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A novel penalty function method for semivectorial bilevel programming problem

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ABSTRACT

The bilevel problem is usually much more difficult to solve than the single level problem. If the bilevel problem can be transformed into a single level problem, it will be much easier for one to design effective algorithms. In this paper, a novel penalty function method for semivectorial bilevel programming problem is proposed. First, based on Benson's method and dual theory of linear programming, the semivectorial bilevel programming problem is transformed into a single level optimization problem by setting the dual gap of the lower level problem to zero; then the transformed problem is further relaxed in order to make the transformed problem solving easier; furthermore, a twice differentiable function called dual gap indicator is designed and a penalty function is constructed by using the dual gap indicator as the penalty term for the relaxed transformed problem. Based on all these, a new penalty function method is proposed, and the solution of the penalty problem can be proved to be the solution of the original problem under mild conditions. Finally, numerical examples are provided to demonstrate the good performance of the proposed algorithm.

1. Introduction

The bilevel programming problem is a hierarchical optimization problem with two levels. It is characterized by the existence of two optimization problems in which a feasible solution of the upper level problem should be an optimal candidate from the lower level problem. Formally, general bilevel programming problem can be given as follows:

$$\min_{x} \quad F(x,y) \text{ s.t. } x \in X, \quad y \in \Psi(x),$$

where $F : R^n \times R^m \to R$, and for each $x \in X \subset R^n$, $\Psi(x)$ is the solution set of the following parametric optimization problem

$$\min_{y} \quad f(x,y) \text{ s.t. } g(x,y) \leqslant 0, \quad y \in Y,$$

where $f : R^n \times R^m \to R$, $g : R^n \times R^m \to R^p$, and Y is a closed set of R^m. The first optimization problem is called the upper level problem and the second one is called the lower level problem.

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The bilevel programming problem is an attractive research field and has been widely applied to network design, transport system planning, management, economics and other aspects [1–5]. In the last two decades, there have been many rich researches on both theoretical results and solution approaches for the bilevel programming problem. For more details on the topic please refer to some reviews [6,7], good surveys [8,9] and useful textbooks [10,11]. However, due to its structure, the bilevel programming problem is very difficult to deal with and has been proved to be NP-hard by Ben-Ayed and Blair [12]. So far the researches on the bilevel programming are still mostly restricted to single objective bilevel cases, and there are only very few studies dealing with multiobjective bilevel cases.

Although multiobjective bilevel optimization problems have not yet received a broad attention in the literature, this class of problem have great potential applications in practice [13]. In this paper, we are interested in the bilevel programming problem with a scalar optimization problem on the upper level and a multiobjective optimization problem on the lower level. This kind of problem was first considered by Bonnel and Morgan [14], and was called semivectorial bilevel optimization problem. Subsequently, Calvete and Galé [15] proposed a genetic algorithm to deal with the bilevel programming problem with multiple linear objective functions at the lower level. Calvete and Galé [16] developed a linear multiobjective bilevel program to model the production–distribution system in a supply chain. Dempe et al. [17] presented the scalarization approach to derive first-order necessary optimality conditions of the optimistic bilevel optimization problem with multi-objective lower level problem.

The penalty function method is an important method for solving bilevel programming problems [18,19]. For semivectorial bilevel optimization problem, Bonnel and Morgan [14] used an exterior penalty method to handle this type of problem in which the objective functions of the upper level and the lower level were defined on Hausdorff topological space. Ankhili and Mansouri [20] addressed an exact penalty method to solve this problem under the assumption that the objective function of the upper level was concave. Another penalty method was developed in [21], which included two different penalty parameters for solving the problem with the linear multiobjective optimization problem on the lower level. In essence, the common point of these two studies is to append the dual gap of the lower level problem to the upper level objective function with a penalty.

However, exact penalty function methods may cause numerical instability in their implementation when the penalty parameter become larger. In practice, we only need to get an approximate solution to the considered problem. Thus the approximation methods of the exact penalty functions have been paid more attention, and some differentiable approximations to the exact penalty functions have been proposed [22–24]. Recently, Wan et al. [25] developed a dual-relax penalty function approach to solve the nonlinear bilevel programming problem with linear lower level problem. It is essential to design an approximation which is first-order differentiable to the exact penalty function.

In this paper, we propose a new penalty function method to solve semivectorial bilevel programming problem with the linear multiobjective optimization problem on the lower level. To be more specific, we first relax the dual gap of the lower level problem to a small positive number, then we design a twice differentiable function called dual gap indicator and construct a penalty function by appending the dual gap indicator to the upper level objective function for the relaxed transformed problem. Based on these, a new algorithm for solving semivectorial bilevel programming problem is proposed.

The rest of this paper is organized as follows. In Section 2, we give the formulation of semivectorial bilevel programming problem and related definitions. In Section 3, we present a penalty function method, provide the main theoretical results, and give the proposed algorithm. In Section 4, we do experiments on test problems to demonstrate the performance of the proposed algorithm. Finally, we conclude the paper in Section 5.

2. Preliminaries

2.1. Optimistic semivectorial bilevel programming

Consider the following bilevel programming problem with a scalar-valued optimization problem on the upper level and a vector-valued problem on the lower level:

$$\min_{x} \quad F(x,y) \text{ s.t. } x \in X, \quad y \in \Psi(x), \tag{1}$$

where $F : R^n \times R^m \to R, X$ is a closed subset of R^n and $\Psi(x)$ is the solution set of the following problem:

$$\min_{y} \quad f(x,y) \text{ s.t. } y \in Y(x), \tag{2}$$

where $f : R^n \times R^m \to R^p$ and $Y(x) \subseteq R^m$ is a set depending on x.

This kind of problem was labeled as semivectorial bilevel optimization problem by Bonnel and Morgan [14]. As we all know, complications arise when there are multiple optima in the lower level problem. As a matter of fact, the upper level problem is not a well defined optimization problem in the case of non-unique lower level solutions. Such an unpleasant situation in some studies can be addressed by assuming that the solution of the lower level problem is uniquely determined for all selections of the upper level decision maker [26]. As far as a multiobjective optimization problem on the lower level is concerned, this assumption is usually not satisfied. In order to make sure that the bilevel problem is well posed, there are at least two approaches available for the upper level decision maker, namely the optimistic approach and the pessimistic