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On the radiative heat transfer in a semi-transparent medium enclosed in a cylindrical annulus of infinite length with specularly reflecting lateral surfaces

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ABSTRACT

In this paper, we present analytical and numerical investigations in order to determine the incident radiation and radiative flux in a semi-transparent medium at radiative equilibrium and enclosed in a cylindrical annulus with two specularly reflecting surfaces. As for the case of a purely diffuse reflexion, we show that the incident radiation and radiative flux can be completely defined by some weighting coefficients whose closed-form analytical expressions are detailed. The specular part of the weighting coefficients is computed with a Gauss quadrature, and a numerical validation is performed by comparing the numerical results to optically thin approximation analytical solutions. Finally some examples show that the results for temperature and radiative flux fields may strongly be different from the ones obtained in the case of a purely diffuse reflexion.

1. Introduction

Radiative transfer in semi-transparent media subjected to multiple boundary conditions in cavities of various geometries is of high technical importance because of its frequent occurrence in numerous industrial applications. An abundant literature deals with media confined in enclosures made of purely diffusely reflecting boundary surfaces for several kinds of geometries, and more specifically for cylindrical geometries [1–7], but enclosures with specularly reflecting surfaces are less examined. The case of the parallel plane slab bounded by specular or diffuse surfaces has been studied [8], and it was noted that for pure radiative transfer, the influence of the reflection nature only slighted affected the behaviour of the radiative field in the determination of the temperature profiles inside the slab. Ganapol [9] determined the radiative field inside an absorbing and scattering semi-infinite medium bounded by a specularly reflecting surface with Fresnel conditions, by solving an integral equation with the help of a Laplace transform. Maruyama [10] studied a plane parallel slab filled with an anisotropic scattering media bounded by reflecting surfaces and compared the radiative fields obtained with diffuse surfaces and opaque specular or transparent Fresnel surfaces. Similar approaches have been developed [11–14] in complex three-dimensional geometries for which boundaries are diffuse and specular. Guo et al. [13,14] used the concept of generalised view factors to take into account the specular reflexion on the boundary surfaces. The case of a specular reflection with Fresnel conditions has been examined for the cylindrical and spherical geometries by means of quasi-analytical

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Nomenclature

$(\vec{e_x}, \vec{e_y}, \vec{e_z})$	unit vectors of the x, y, z directions
G	volumic incident radiation (W m^{-2})
$I_S(\vec{\Omega})$	intensity leaving a boundary surface for a given direction $\vec{\Omega}$ (W m^{-2} Sr^{-1})
Ki_n	Bickley–Naylor functions
N	cells number between the two cylinders
$\vec{q}^r$	radiative flux vector (W m^{-2})
T	temperature (K)
x, y, z	coordinate axis directions

Greek letters

Δr	characteristic cell length
κ	absorption coefficient (m^{-1})
ε	emissivity of the cylinders surfaces
ρ	reflection factor of the cylinders surfaces
σ	Stefan–Boltzmann constant (5.6710^{-8} W m^{-2} K^{-4})
φ, θ	angular description of the unit vector $\vec{\Omega}$
$\vec{\Omega}$	unit vector of radiation propagation

Subscripts

i, e	inner and outer cylinder walls, respectively

developments. Cassell et al. [15,16] developed a general integral equation inside a cylinder bounded by a specularly reflecting surface, which allows the determination of the intensity field when the medium is anisotropically scattering.

A similar work was performed inside a two regions sphere, the two boundaries being subjected to Fresnel conditions. Tian et al. [17] examined the case of a quasi-isothermal hollow cylinder of infinite length with the help of a ray tracing method, and compared the equivalent radiative absorption obtained in this case and the one of diffuse surfaces. Chakravarthy et al. [18] extended the previous study to an eccentric cylinder annulus with Fresnel surface by using a Monte-Carlo ray tracing technique. Wu et al. [19] applied the discrete ordinates method (DOM) in a two-dimensional cylindrical enclose of finite length with ending Fresnel specularly reflecting surfaces, by taking into account the presence of a critical angle due to the refraction at the interface, and compared their technique with a Monte-Carlo method. Rukolaine [20] and Budenkova [21] applied the DOM with an improved numerical scheme to treat the specular reflexion in irregular enclosures with opaque and transparent both diffuse and Fresnel surfaces. In [21] a mirror-image method combined to analytical developments allowed to study radiative transfer in cylindrical and conical enclosures with Fresnel reflecting surfaces. Alternative approaches have been proposed by Hunt [22] who obtained the source from a Bessel Integral Transform giving the asymptotic trend of the intensity for large values of the radius of the cylinder.

Altaç [23] completely transformed the angular integration appearing in the incident radiation to a spatial integration in the case of a plain cylinder of infinite length. It is worth noting that this transformation of angular integrals into spatial ones completely eliminates the ray effect and is well adapted to enclosures with diffuse boundaries. In a previous paper [24] we extended the method developed by Altaç by keeping a hybrid formulation which handles both the spatial and angular description, in the case of a cylindrical annulus with diffuse reflecting surfaces. The aim of this present work is to take into account a specular reflection for a cylindrical annular geometry.

The paper is organised as follows: in Section 2, we shortly describe the physical system and remind the exact expressions of the incident radiation and radiative flux inside the medium, which take into account the boundary conditions on the cylinders lateral surfaces. The determination of the boundary intensities leaving the two cylinders surfaces for specularly reflecting surfaces is completely detailed in Section 3, and the useful discrete form of these boundary intensities is developed in Section 4. Sections 5 and 6 are finally devoted to the presentation of calculation details and some results which assess the correctness of the present method.

2. Expression of the radiative field inside the annulus

One considers a cylindrical annulus of infinite length, filled with an absorbing-emitting but non scattering semi-transparent gray medium at radiative equilibrium, of absorption coefficient κ and unit refractive index not depending on the internal temperature field. The boundary surfaces of the two concentric cylinders are assumed opaque, isothermal with imposed temperatures, and specularly reflecting. This means that a reflected ray at one point on the lateral surface obeys to the Descartes' law, the reflected angle being identical to the incident one relatively to the normal vector to the tangent plane of the surface at the reflexion point. The incident radiation and the radiative flux at an internal point write: