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Finite-time H_∞ control for singular Markovian jump systems with partly unknown transition rates

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ABSTRACT

This paper investigates the problem of finite-time H_∞ control for a class of singular Markovian jump systems with partly unknown transition rates. Firstly, sufficient conditions on singular stochastic finite-time boundedness of singular Markovian jump systems with partly unknown transition rates are obtained. Secondly, the results are extended to singular stochastic H_∞ finite-time boundedness of singular Markovian jump systems with partly unknown transition rates. Then state feedback controllers are designed to ensure the singular stochastic finite-time boundedness and the singular stochastic H_∞ finite-time boundedness of the underlying closed singular Markovian jump systems in the forms of strict LMIs. Finally, numerical examples are given to illustrate the validity of the proposed methods.

1. Introduction

Singular systems, which are also referred to as generalized systems, descriptor systems, implicit systems, or differential–algebraic systems, have attracted many researchers due to the fact that singular systems have extensive applications in many practical systems such as electrical circuits, power systems, networks and other systems [1,2]. In recent years, many research topics on singular systems have been extensively studied such as stability and stabilization [3,4], H_∞ control problem [5,6], and so forth. A lot of attention has been paid to the study of Markovian jump systems (MJSs), which are a special kind of hybrid systems, because they have the advantage of better representing physical systems with random changes in their structures and parameters, and have successful applications in economic systems, manufacturing systems and communication systems [7–9]. Many important issues have been studied for this kind of systems, such as stability analysis, stabilization, H_2 and H_∞ control [10–13]. When singular systems experience abrupt changes in their structures, it is natural to model them as singular Markovian jump systems (SMJSs) [14–16].

On the other hand, in most of the studies, complete knowledge of the mode transition rates is required as a prerequisite for analysis and synthesis of MJSs. Therefore, rather than having a large complexity to measure or estimate all the transition rates, it is significant and necessary to further study more general jump systems with partly unknown transition rates. Many attracting problems have been investigated and solved for these systems, such as stability analysis, stabilization [17,18] and H_∞ filter [19–21]. In many practical applications, one may be interested in not only the stability of the system but also the

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behavior of the system over a fixed short time. For instance, large values of the state are not acceptable in the presence of saturations [22,23]. Compared with the classical Lyapunov asymptotical stability, in order to deal with these transient performances of control dynamic systems, finite-time stability or short-time stability was introduced. Some appealing results were obtained to ensure finite-time stability, finite-time boundedness and finite-time stabilization of various systems including linear systems, nonlinear systems, and stochastic systems [24–28]. To the best of our knowledge, the problem of singular stochastic finite-time H_∞ control for singular Markovian jump systems (SMJSs) with partly unknown transition rates has not been fully investigated, which motivates the main purpose of our study.

In this paper, we are concerned with the problem of finite-time H_∞ control for SMJSs with partly unknown transition rates. And SMJSs with completely known transition rates are viewed as special cases of the ones tackled here. Firstly, we give the concepts of singular stochastic finite-time boundedness (SSFTB) and singular stochastic H_∞ finite-time boundedness (SSH_∞FTB) of SMJSs. The main contribute lies in tractable sufficient conditions obtained to guarantee SSFTB and SSH_∞FTB of SMJSs with partly unknown transition rates. Then, a finite-time H_∞ state feedback controller is designed in the form of LMIs, which ensures SSH_∞FTB of the closed-loop system. Numerical examples are provided to demonstrate the effectiveness of the main results.

Notation: Throughout this paper, the notations used are fairly standard, for real symmetric matrices A and B, the notation $A \geqslant B$ (respectively, $A > B$) means that the matrix $A - B$ is positive semi-definite (respectively, positive definite). A^{T} represents the transpose of a matrix A, and A^{-1} represents the inverse of a matrix A. $\lambda_{\max}B$ ($\lambda_{\min}B$) is the maximum (respectively, minimum) eigenvalue of a matrix B. diag$\{\cdots\}$ stands for a block-diagonal matrix. I is the unit matrix with appropriate dimensions, and in a matrix, the term of symmetry is stated by the asterisk '$*$'. Let $\mathbb{R}^n$ stands for the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all real matrices, and $\|\cdot\|$ stands for the Euclidean norm of vectors. $\mathcal{E}\{\cdot\}$ denotes the mathematics expectation of the stochastic process or vector. $L_2^n[0,\infty)$ stands for the space of n-dimensional square integrable functions on $[0,\infty)$.

2. Basic definitions and lemmas

Consider a class of SMJSs as follows:

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) + B_w(r_t)w(t) \\ z(t) = C(r_t)x(t) + D(r_t)u(t) + D_w(r_t)w(t) \end{cases}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $z(t) \in \mathbb{R}^p$ is the controlled output, $E \in \mathbb{R}^{n\times n}$ with rank$E = r \leqslant n$; $w(t) \in \mathbb{R}^q$ is the disturbance which belongs to $L_2^q[0,\infty)$, and satisfies

$$\int_0^T w^{\mathrm{T}}(t)w(t)\mathrm{d}t \leqslant d^2, \quad d \geqslant 0, \tag{2}$$

$A(r_t), B(r_t), B_w(r_t), C(r_t), D(r_t)$ and $D_w(r_t)$ are real known matrices with appropriate dimensions. $\{r_t, t \geqslant 0\}$ is a continuous-time Markov process with right continuous trajectories taking values in a finite set given by $\mathcal{S} = \{1, 2, \ldots, N\}$ with the transition rate matrix (TRM) $\Pi \triangleq \{\pi_{ij}\}$ given by

$$\Pr\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ii}h + o(h), & i = j \end{cases},$$

where $h > 0, \lim_{h\to 0}\frac{o(h)}{h} = 0$ and $\pi_{ij} \geqslant 0$, for $j \neq i$, is transition rate from mode i to j at time $t+h$, which satisfies $\pi_{ii} = -\sum_{j=1, j\neq i}^{N}\pi_{ij}$. The transition rates of the jumping process may be considered to be partly accessible in this paper. For instance, the TRM of system (1) may be expressed as

$$\Pi = \begin{bmatrix} \pi_{11} & \hat{\pi}_{12} & \cdots & \hat{\pi}_{1N} \\ \pi_{21} & \hat{\pi}_{22} & \cdots & \pi_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\pi}_{N1} & \pi_{N2} & \cdots & \pi_{NN} \end{bmatrix},$$

where each unknown element is labeled with a hat '$\hat{\ }$'. For notational clarity, $\forall i \in \mathcal{S}$, the set $U^{(i)}$ denotes $U^{(i)} = U_k^{(i)} \cup U_{uk}^{(i)}$ with $U_k^{(i)} \triangleq \{j | \pi_{ij}$ is known for $i \in \mathcal{S}\}, U_{uk}^{(i)} \triangleq \{j | \pi_{ij}$ is unknown for $i \in \mathcal{S}\}$. Moreover, if $U_k^{(i)} \neq \phi$, it is further described as

$$U_k^{(i)} = \{k_1^{(i)}, k_2^{(i)}, \ldots, k_{m_i}^{(i)}\}, \quad m_i \in \{1, 2, \ldots, N-2\}, \tag{3}$$

where $k_j^{(i)} \in Z^+$, $1 \leqslant k_j^{(i)} \leqslant N$, $j = 1, 2, \ldots, m_i$ represents the jth known element of the set $U_k^{(i)}$ in the TRM Π. We denote $\pi_k^{(i)} \triangleq \sum_{j\in U_k^{(i)}}\pi_{ij}$, and when $\hat{\pi}_{ii}$ is unknown, it is necessary to provide a lower bound $\pi_d^{(i)}$ for it.

Consider a state feedback controller

$$u(t) = K(r_t)x(t), \tag{4}$$