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A near-wall domain decomposition approach in application to turbulent flow in a diffuser

Adam Jones [*], Sergey Utyuzhnikov

School of Mechanical, Aerospace and Civil Engineering, University of Manchester, Manchester M13 9PL, UK

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ABSTRACT

Turbulent flow in an asymmetric, two-dimensional diffuser is studied using a near-wall domain decomposition method and a k—ε turbulence model. A one-dimensional boundary layer equation is used to transfer the boundary conditions from the wall to an interface within the flow. The boundary conditions applied to the fluid velocity and turbulent kinetic energy are of Robin type. They are mesh independent and can account for arbitrary source terms. This approach avoids the computational expense of fully simulating the turbulent boundary layers. For the first time, the technique has been applied to modelling a separated flow with an unstructured code. It is shown how the interface boundary condition on the turbulent kinetic energy allows the recirculation region in the diffuser to be captured. In contrast, the standard wall function approach, based on the log law, fails to predict any recirculation region. The only parameter required to apply the domain decomposition method is a turbulent viscosity profile across the boundary layer. Three different profiles are used in this work. It is shown how making the turbulent viscosity a function of the pressure gradient improves flow predictions for the diffuser. The results demonstrate that the method is an efficient way to simulate the boundary layers in engineering problems that include complex geometries or separating flows.

1. Introduction

Simulation of wall-bounded turbulent flows is computationally expensive because it requires resolution of turbulent boundary layers. These thin, near-wall regions next to the laminar sublayer are always present because of the no-slip boundary condition and the damping effect of the wall. A fine mesh is required to capture the large gradients that occur in these regions. The structure of turbulent boundary layers can be resolved with low Reynolds number (LRN) turbulence models, whose governing equations remain valid to the wall. The name comes from the low turbulent Reynolds number in the boundary layer. Damping functions are often introduced into the turbulence equations to generate the appropriate wall-limiting behaviour of each function. However, resolution of the turbulent boundary layers can account for over 90% of the total run time of a simulation, which makes LRN models unappealing for industrial applications.

The alternative is not to fully resolve the boundary layer. This is the approach taken by high Reynolds number (HRN) models, which typically utilise wall functions. Wall functions use empirical correlations or results from simplified test cases to compute boundary conditions at the wall that account for the variation in the flow across the boundary layer. The

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[*] Corresponding author. Tel.: +44 161 306 3707.
E-mail address: adam.jones-2@manchester.ac.uk (A. Jones).

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empirical correlations are also used to compute any other required terms, such as the production of turbulent kinetic energy. With such an approach, the computational mesh can be coarse near to the wall, which makes HRN models more efficient than LRN models. However, typically their accuracy is reduced and often the assumptions upon which wall functions are derived only apply in certain types of flow. As noted in [1], the wall function should represent the inner region in domain decomposition.

The earliest wall function was based on the logarithmic law of the wall, which assumes that the turbulence is in local equilibrium in the turbulent region of the boundary layer. To apply this wall function, the first near-wall computational node must lie within the fully turbulent region. This requirement is often impossible to satisfy in three-dimensional flows and leads to mesh-dependent solutions. The scalable wall function [2] is an early attempt to improve the range of validity of the log law. In this approach, if the first computational node lies within the viscous sublayer, the boundary conditions are applied as if the computational node were at the edge of the viscous sublayer. However the wall function lacks generality; a general wall function must be able to handle source terms in the momentum equations.

A more sophisticated wall function is the analytical wall function [3]. This is based upon analytical integration of simplified boundary-layer equations over each near-wall cell. It does not use the log law assumption. Integration is made possible by assuming that the turbulent viscosity varies linearly from the edge of the viscous sublayer up to the far edge of the near-wall cell. Another development is the numerical wall function [4], which solves a one-dimensional transport equation over a near-wall sub-grid that spans the near-wall cell. Convection and low-Reynolds-number terms can be included in the governing equations. Both approaches specify a Dirichlet boundary condition at the first near-wall cell, which is updated at each iteration with information from the mean flow. This can be interpreted as a domain decomposition approach, with one domain limited by the centre of the cell nearest to the wall. The analytical and numerical wall functions predict complex flows with more accuracy than the log law, however they have not been widely adopted in industry because implementation of them into industrial codes requires significant changes to the code and does not generalise well to unstructured solvers.

The compound wall treatment [5] uses a one-dimensional boundary layer equation and includes source terms as a single parameter that is assumed constant over the sublayer. The solutions in the viscous and turbulent region are blended together to make the wall function valid at all points in the boundary layer. Although easy to implement and robust for industrial applications, this approach lacks validity and accuracy in complex flows, owing to its underlying assumptions.

Another class of wall function uses look-up tables for the wall shear stress. The look-up table is generated by solving simplified boundary layer equations in the absence of source terms [6]. The principle of “wall layer universality” underpins this method, which does not, in general, hold. Furthermore it is unclear how the method can be extended to handle source terms.

This work uses the theory of interface boundary conditions (IBCs) [7–11]. This approach was first based on the same assumptions as the analytical wall function [3], but has been developed into a domain decomposition method which is applicable for LRN turbulence models.

To derive IBCs, a one-dimensional boundary layer equation is assumed to hold over the near-wall region, with Dirichlet or Neumann conditions applied at the wall. The boundary layer equation is used to transfer the boundary conditions from the wall to a location at an interface above the wall, within the fluid domain. The result is always a boundary condition of Robin type at the interface. The Robin boundary condition removes the need to compute the near-wall region of the flow. All flow variables, including velocities, scalars and turbulence functions can be treated with this approach. This produces an appealing, unified treatment. The only free parameter in the method is the turbulent viscosity profile, which is applied in the near-wall boundary layer equation. Once a converged solution is reached, the solution across the boundary layer can be found by a separate calculation, if it is required.

Wall functions are often formulated in terms of mesh parameters such as the volume of the near-wall cell [3,4]. In contrast, IBCs are derived in a mesh-independent form and source terms can be incorporated in the boundary layer equation.

IBCs have been successfully applied to the case of a one-dimensional channel flow with two different viscosity profiles [8,11]. In each case the results show little sensitivity to the distance from the wall. The method has also been applied in a structured code to a two-dimensional impinging jet flow [9,10].

For flows in complex geometries, non-local effects can be important. IBCs can be generalised into a non-local formalism via the theory of Calderón–Ryaben’kii potentials [10]. This is outlined for a two-dimensional model equation in [12].

There is a trade-off between accuracy and efficiency with all near-wall modelling. A fully resolved LRN solution will always be accurate to the maximum extent possible with the chosen turbulence model, whereas a HRN solution with the log law-based wall function will produce inaccurate results in all but the simplest of flows. However, the LRN solution may require an order of magnitude more CPU time to converge [4]. IBCs are appealing because complex physics can be included in the governing equations and a good enough solution obtained across the boundary layer for a small computational cost.

LRN turbulence models can be used with IBCs [11]. In such cases, as the interface approaches the wall, the IBCs tend to the usual wall boundary conditions. Alternatively, if the interface is sufficiently far away from the wall then a HRN model emerges. The meshes in the inner and outer regions are independent. Hence mesh generation and mesh independence studies are simpler with IBCs than with conventional wall functions.

In this paper, IBCs have been implemented into an unstructured code for the first time. The chosen code is *Code_Saturne*, which is an open-source, industrial code developed by EDF R&D. For the first time, IBCs have been applied to a separating flow in a complex geometry. The method has been applied to a test case of an axisymmetric two-dimensional impinging