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Application of radial basis functions to the problem of elasto-plastic torsion of prismatic bars

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ABSTRACT

This paper demonstrates the use of a well-known meshless method, radial basis functions (RBF), to solve the torsion of a prismatic bar having a rectangular/square cross-section. First part of the analysis deals with the elastic solution of the problem formulated using the RBF technique. The result is used in verifying the feasibility of the approach and, subsequently, as an initial guess for the iterative procedure utilized in the analysis of the elasto-plastic torsional behavior of the bar. Verification of the results is made using finite difference method (FDM), method of fundamental solutions (MFS) and the exact elastic solution.

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1. Introduction

1.1. Analysis of elasto-plastic torsion

Various studies have reported the application of different techniques to solve elasto-plastic torsion problems. Earlier solutions of the problem were obtained based on sand heap analogy [1,2] and a combination of membrane analogy and sand heap analogy [3]. An approach using non-linear programming was used to obtain elasto-plastic torsion of perfectly plastic material by Hodge [4,5] and by Hodge et al. [6]. Billinghurst et al. [7] developed a mitre model for the shear strain distribution in steel members under uniform torsion. The model uses a simple approximation to the shear strain distribution caused by uniform torsion. In addition to accurate predictions of the fully plastic uniform torque, acceptable approximations for the shear stress distributions were achieved (except near re-entrant corners).

Numerical approaches for the solution of elasto-plastic torsion problem have gained recognition due to their flexibility and capability to tackle complex equations whose analytical solutions do not exist. Finite difference method (FDM) was applied to obtain a solution for elasto-plastic torsion by Christpherson for an I-section [8], by Dwivedi et al. [9] for the solution of a torsional springback in square bar, and solution by FDM compared against that of non-linear programming by Hodge et al. [6]. Application of finite element method (FEM) to study the elasto-plastic torsion was first carried out by Yamada et al. [10]. Subsequently, several other works on elasto-plastic torsion analysis using FEM are reported [11–14]. More recently, Kucwaj [15] presented the application of a remeshing algorithm to solution of elastic–plastic torsion of bars with isotropic strain hardening. The remeshing algorithm uses a grid generator with mesh size function. The optimal mesh size for the posed problem is obtained iteratively. Boundary element method (BEM), a mesh reduction method that requires boundary-only discretization, was applied by Sapountzakis and Tsipiras [16] to analyze non-linear elasticplastic uniform

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torsion problem. MFS was used by Kolodziej and Gorzelanczyk [17] to analyze both elastic and elasto-plastic torsion of bars. The non-linear torsion problem in plastic region was solved by means of the Picard iteration.

As is widely known, the difficulty, accuracy and/or rate of convergence in solving any set of partial differential equations vary depending on the proper choice of the solution scheme. With the exception of BEM and MFS, all the aforementioned numerical methods applied to obtain solutions of elasto-plastic torsion require discretization of both the domain and boundary of the problem. However, in order to decrease the computational effort with no compromise to the accuracy of the solution, in addition to some other obvious advantages, it would be preferable to avoid discretizing the problem. This results in an increase in popularity of the meshfree methods (such as RBF). Even the BEM utilized in [16] requires that the boundary be discretized and, hence, is not purely meshless. Although studies such as [17] utilize the RBF as an interpolant of the non-homogeneous term of the governing equation, the fact still remains that they use this approach in conjunction with another to obtain the overall solution. For instance, in Ref. [17] MFS is used as the main solution technique with the RBF used only to interpolate the right hand side of the governing equation. Hence, the study presented in this paper makes use of the RBF alone in solving elasto-plastic torsion of bars and the result verified against an FDM solution and that reported in [17]. As will be seen, the RBF approach results into an enormous reduction in the computational effort compared to that using MFS. Another major advantage of using the RBF alone in this study is the simplicity of the problem formulation and solution procedure presented herein.

1.2. RBF-based collocation methods

Application of meshless methods in solving solid and structural mechanics problems is becoming increasingly popular especially with the growing possibilities of using fast computing facilities to enable their implementation. RBF is one of the most famous types of such meshfree methods and it belongs to a class of functions that depend only on the Euclidean distance, $r = |\bar{x} - x_k|$ of a domain point $\bar{x}(x, y)$ from a variable domain point $x_k(x_k - y_k)$ and is of the form $\phi(r)$. A comprehensive review of recent RBF collocation methods is available in [18]. The first attempt to use RBF for solving partial differential equations was made by Kansa in the early 1990s [19,20]. The RBF in his method is the multiquadrics (MQ) type which proves to be efficient if an appropriate shape parameter is chosen. Sequel to the work in [19], benefits of using MQ were further investigated and used as the spatial approximation scheme for parabolic, hyperbolic and the elliptic Poisson's equation in [20]. Illustrations were made on its accuracy and higher efficiency than the finite difference schemes.

As reported in the literature [18], the numerical accuracy of RBF solutions depends on the grid size, the shape parameter, the nature (in terms of complexity) of the functions involved and some other potential factors. As a result, some works have been carried out in order to find the relationship between accuracy of the RBF and the factors affecting it [21–25]. Ways of estimating the error due to RBF interpolation are reported in [26–31].

The matrix resulting in the formulation of RBF collocation scheme in [19,20] is unsymmetric. Hence, Hermite collocation method [32] is invented to tackle such deficiency. However, solutions by the Hermite collocation method lack accuracy in the regions adjacent to the boundary. Consequently, Chen [33] made use of Green second identity and presented a modified Kansa method (MKM) that is symmetric and improves the accuracy of the solution in the vicinity of the boundary.

The foregoing discussion on different forms of RBF collocation schemes are based on global approximation: The RBF interpolation is by all the collocation points in the whole physical domain and boundary. This results in fully populated matrices. In cases where the matrices are ill-conditioned, use of localized RBF methods is an excellent alternative. Regularization of the ill-conditioning of dense matrix can be achieved using the singular value decomposition (SVD) [34] as proved in [35–37]. Other viable alternative approaches are reported in [38–40,23].

The BEM is another popular meshless method (or more appropriately referred to as mesh reduction method). It consists of boundary only discretization without the need for domain discretization. This results in a remarkable reduction in the number of elements/mesh and computations required, hence the name mesh reduction method.

One major setback of BEM is the situation where the differential equations have an inhomogeneous term and domain integration is required. In such cases, use of RBF to circumvent this limitation is very effective. Some of the previous studies that employed the use of RBF as an interpolation function to approximate the nonhomogeneous term include those reported in Refs. [41–47].

Numerical integration is required in the BEM, and since this, in turn, requires meshing the boundary, the BEM is believed not to be truly meshless. This motivates many works to be done in extending the power of RBF to tackle problems using boundary-only distribution of nodes. These techniques are termed the boundary-type RBF collocation methods. They are integration-free, easy-to-use, and truly meshless. One of the popular boundary-type RBF collocation methods is MFS [48]. MFS requires a distribution of the source nodes on fictitious boundary outside the physical domain to avoid the singularities of fundamental solutions. However, its practical applicability is sometimes limited in cases of complex-shaped boundaries and multiple connected domain problems. To overcome such limitations, another boundary-type RBF collocation method, the boundary knot method (BKM) [49] was proposed. In BKM, the singular fundamental solutions are replaced by nonsingular RBF general solutions. However, as a priori the general solution of the problem at hand is needed in BKM. In addition, the ill-conditioned matrices are sometimes encountered. As a result, several other forms of boundary-type RBF collocation methods are proposed as alternatives [50–55]. Evaluation of the particular solution for non-homogeneous problems requires combining the boundary-type RBF collocation methods with other techniques such as the dual reciprocity method (DRM) [56] and multiple reciprocity method (MRM) [57].

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