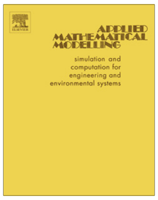
Applied Mathematical Modelling 40 (2016) 451–467

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm



Non-linear dynamic response to simple harmonic excitation of a thin-walled beam with a breathing crack

Franco E. Dotti, Víctor H. Cortínez *, Florencia Reguera

Centro de Investigaciones en Mecánica Teórica y Aplicada, Universidad Tecnológica Nacional, FRBB, 11 de Abril 461, B8000LMI Bahía Blanca, Argentina
Consejo Nacional de Investigaciones Científicas y Técnicas, CONICET, Argentina

ARTICLE INFO

Article history:
Received 21 January 2014
Received in revised form 11 February 2015
Accepted 22 April 2015
Available online 9 June 2015

Keywords:
Thin-walled beams
Breathing crack
Period doubling
Quasi-periodicity

ABSTRACT

The dynamic response of a thin-walled beam with a breathing crack is studied by employing a refined one-dimensional model introduced for such purpose. It is shown that, due to the non-linearity of breathing, some effects take place which are impossible by using a completely open crack model. Even with the simplest of sinusoidal excitations, the system under study reveals a rich and complex dynamics. Some of the topics emphasized in the article are self-excitation of harmonic resonances, period doubling and the presence of quasi-periodic motion. Furthermore, the possibility of chaotic vibrations is analyzed.

1. Introduction

The presence of damage represents a crucial topic regarding safety of engineering structures. Since early damage detection may prevent the possibility of service failures, the development of structural health monitoring techniques has acquired a remarkable importance in recent years [1–3]. A very common damage typology that appears in dynamic operating condition corresponds to fatigue cracks. These cracks are usually invisible to the naked eye and their identification by local non-destructive methods, such as dye penetrant and ultrasonic testing, may be impractical if the structure must remain in service.

These aspects lead to the need of mathematical models as interpretative tools for crack identification, which is generally performed by solving an inverse optimization problem [4]. This problem is usually complex from a mathematical point of view, and also must be solved quickly. Hence models must combine precision and simplicity in balanced form. For slender structures, a beam model captures the most significant features of the structural dynamics. And at the same time it is simple enough for extensive computational treatment, an essentially useful feature when damage diagnosis must be performed in real time. Most studies have focused on Bernoulli–Euler beams [5–9] and, to a lesser extent, on Timoshenko beams [10–13]. Thin-walled beams have not been addressed in depth, perhaps due to their more complex dynamics, usually presenting a strongly coupled vibrational response [14–22].

Crack models are generally assumed to be linear, with a crack remaining open during vibration. But while this hypothesis works well for many situations, it is more realistic to consider a fatigue crack which may be open or closed during the motion [10,23,24]. This "breathing effect" introduces a non-linearity in the system since the stiffness of the structure becomes time

---

* Corresponding author at: Centro de Investigaciones en Mecánica Teórica y Aplicada, Universidad Tecnológica Nacional, FRBB, 11 de Abril 461, B8000LMI Bahía Blanca, Argentina. Tel.: +54 291 4555220; fax: +54 291 4555311.

E-mail address: vcortine@frbb.utn.edu.ar (V.H. Cortínez).

http://dx.doi.org/10.1016/j.apm.2015.04.052
0307-904X/

Nomenclature

A	crack depth (also semi-major axis of the elliptic crack)
$\tilde{a}$	variable crack depth
A	cross-sectional area
A_f	loading amplitude
b	dimension of a flange
B	bimomental beam force (also point B, origin of the system B: x, s, n)
c	semi-minor axis of the elliptic crack
C	center of gravity of the uncracked cross section
C_w	warping constant
$\mathbf{D}$	damping matrix
E	Young's modulus
f	driving frequency, expressed in Hz
f_B	bilinear frequency, expressed in Hz
$\mathbf{F}_C^{(e)}$	flexibility matrix of the cracked element
G	shear modulus
G^*	crack mouth widening energy release rate
h	dimension of the web
H	point placed at the cracked branch of the cross-section
I_y, I_z	second moments of area
I_{yz}	product moment of area
$I_{y\omega}$, $I_{z\omega}$	product of warping
I_{sv}	Saint–Venant's torsion constant
I_{ij}	shear coefficients of the constitutive matrix
J	Rice's J integral
$\mathbf{J}$	constitutive matrix of the beam
K	point placed at the cracked branch of the cross-section
$\mathbf{K}_0$	global stiffness matrix of an intact beam
$\mathbf{K}_C$	global stiffness matrix of a beam with a fully open crack
$\mathbf{K}_B$	global stiffness matrix of a beam with a breathing crack
$\mathbf{K}_C^{(e)}$	stiffness matrix of the cracked element
L	length of the beam
M_x	total torque
$\mathbf{M}$	inertia matrix of the uncracked beam
n	coordinate normal to the cross-section middle line
N	axial beam force
M_y, M_z	bending moments
$\mathbf{p}^{(i)}$	vector of nodal forces of node i
P_i	nodal forces
$\mathbf{Q}$	vector of generalized forces
r	distance in s-direction from C to a point in the cross-section middle line
s	circumferential coordinate
S	cross-sectional perimeter
S_y, S_z	first moments of area
S_ω	first moment of warping
t	wall thickness
T_f	driving period
$\mathbf{T}$	transformation matrix
T_{sv}	Saint–Venant torque
T_w	Vlasov torque
u, v, w	displacements of the uncracked beam centroid
u_x, u_y, u_z	displacements of any point of the beam
U	strain energy of the beam
U_0	strain energy of the uncracked element
U_C	strain energy due to the presence of a crack
V	work of external loads
$\mathbf{w}^{(i)}$	vector of nodal displacements of node i
x, y, z	Cartesian coordinates
$\bar{x}_C$	location of the crack in the cracked element
Y, Z	coordinates of a point located in the middle line of the cross-section