Applied Mathematical Modelling 40 (2016) 500–509

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

New visualization method for vortex structure in turbulence by lambda2 and vortex filaments

Yinlin Dong, Yonghua Yan, Chaoqun Liu *

University of Texas at Arlington, Arlington, TX, USA

ARTICLE INFO

Article history:
Received 15 July 2014
Received in revised form 8 March 2015
Accepted 17 April 2015
Available online 25 June 2015

Keywords:
Vortex structures
Visualization
λ_2 method
Vortex filaments

ABSTRACT

In this paper, a new visualization method for vortex structure is presented. Several widely used techniques for local vortex visualization in turbulent flows are reviewed and compared. Among these techniques, lambda2 and *Q* method are the most prominent and they are considered to be the best methods in visualizing vortical structures. However, as we illustrate, there are some defects in those iso-surface based methods. They may cause fake vortex breakdown since they can only capture the strong rotation centers and may not reveal the true vortex structure, i.e., the λ_2 iso-surface may only represent rotation cores but not necessarily vortex tubes. To better illustrate the real vortical structures and distinguish the vortex, which has a rotation core but vorticity lines can penetrate, from the vortex tubes, which vorticity lines cannot penetrate, we present a new visualization method via a hybrid of lambda2 and vortex filaments. As shown in our examples, the new method enables us to find the accurate vortex structure and well capture the physics.

1. Introduction

One of the great challenges in fluid dynamics is to gain a comprehensive understanding of turbulent flows. A well-established approach is the analysis of the vortical structures contained in the flow. As has been shown both experimentally and numerically [1,2], vortical structures play an essential role in the turbulence dynamics such as turbulence generation, kinetic energy production and dissipation, enhancement of transport of mass, heat and momentum and so on [3,4]. It is therefore important to understand the generation, interaction and evolution mechanisms of vortical structures.

Although up to now there is no generally acceptable definition for vortices, it does not bother us to study these basic flow-field structures from visualization. Detection and visualization of vortices is important since they carry most of the energy of vortical flows and thus contribute considerably to the flow evolution [5–7]. Using visualization techniques one should be able to observe and track the formation, convection and evolution of vortices in fluid flow transition and turbulent boundary layer [2,8]. Various methods of identification and visualization of vortices have been proposed so far [9–13].

The common intuitive measures of vortex identification are inadequate. As has been discussed in Jeong and Hussain [11], first, vorticity which is one of the most natural candidates for the characterization of vortical motions does not distinguish between shear layers and the swirling motion of vortices. The geometrical structure of the iso-surfaces of the vorticity magnitude varies with the chosen value. Second, the existence of a local pressure minimum is neither a sufficient nor a necessary condition for the presence of a vortex. Third, the use of spiraling streamlines or pathlines is also problematic because they are

* Corresponding author.

http://dx.doi.org/10.1016/j.apm.2015.04.059
0307-904X/

not Galilean invariant, i.e., independent of the transitional velocity of an observer. Currently the most widely used techniques are the local methods for vortex identification based on the analysis of the velocity gradient tensor $\nabla\mathbf{u}$ such as Δ method, Q method, swirling strength and λ_2 method. These methods can give good results in many situations, but, on the other hand, all of them can be shown to be failed or at least to provide ambiguous answers in certain circumstances [14,15]. Similar concerns have been given by others [16,17].

In Section 2, we will give a brief review and comparison of the local visualization methods. In Section 3, we point out that lambda2 is an iso-surface and can give fake vortex breakdown and other non-physical vortex structures by improper λ_2 values. We present a new method for vortex identification and visualization by capturing the vortex core first with proper λ_2 and then tracking the vortex filaments to find the origin. In Section 4, several illustrative examples are given and discussed. Conclusions are drawn in Section 5.

2. Local vortex visualization techniques

The most widely used local vortex visualization methods are based on the analysis of the velocity gradient tensor $\nabla\mathbf{u}$. Let $\mathbf{u}$ be a 3D velocity field from which vortices are to be extracted. For every grid point of this vector field, $\nabla\mathbf{u}$ is then computed and decomposed into a symmetric rate-of-strain tensor $S = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)/2$ and an antisymmetric rate-of-rotation tensor $\Omega = (\nabla\mathbf{u} - \nabla\mathbf{u}^T)/2$, i.e.,

$$S = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad \Omega = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right).$$

2.1. Δ method

Using critical point theory Chong et al. [9] defined a vortex core to be the connected region in which $\nabla\mathbf{u}$ has complex eigenvalues. In a non-rotating reference frame translating with a fluid particle, the instantaneous streamline pattern obtained from Taylor series expansion of the local velocity to a linear order is governed by the eigenvalues of $\nabla\mathbf{u}$. For both compressible and incompressible flows, these streamlines are closed or spiraling if two of the eigenvalues form a complex conjugate pair.

The eigenvalues λ of $\nabla\mathbf{u}$ satisfy the characteristic equation $\lambda^3 - P\lambda^2 + Q\lambda - R = 0$, where P, Q and R are the three invariants of $\nabla\mathbf{u}$, defined by

$$P = \nabla\cdot\mathbf{u}, \quad Q = \frac{(\nabla\cdot\mathbf{u})^2 - tr(\nabla\mathbf{u})^2}{2} \text{ and } R = \det(\nabla\mathbf{u}).$$

For an incompressible flow, $\nabla\cdot\mathbf{u} = 0$. The discriminant of the characteristic equation is given by $\Delta = (Q/3)^3 + (R/2)^2$. Complex eigenvalues of the velocity gradient tensor occur when the discriminant $\Delta > 0$.

2.2. Q method

Hunt et al. identified vortices as flow regions with positive second invariant of $\nabla\mathbf{u}$ [11], i.e., $Q > 0$. In addition, the pressure in the vortex region is required to be lower than the ambient value. For an incompressible flow, Q can be written as

$$Q = \frac{1}{2}\left(\|\Omega\|^2 - \|S\|^2\right).$$

According to this method, a region of vorticity appears as a vortex if the local rate-of-rotation is larger than the rate-of-strain, i.e., the antisymmetric part of $\nabla\mathbf{u}$ is prevailing over the symmetric part. It can be seen that the Q method is more restrictive than the Δ method.

2.3. Swirling strength method

Zhou et al. [13] used the imaginary part of the complex eigenvalue of $\nabla\mathbf{u}$ to visualize vortices and to quantify the strength of the local swirling motion inside the vortex. When $\nabla\mathbf{u}$ has complex eigenvalues, they can be donated by $\lambda_{cr} \pm i\lambda_{ir}$. The 'swirling strength', given by λ_{ci}, is a measure of the local swirling rate inside the vortex. The strength of stretching or compression is determined by the real eigenvalue λ_{cr}. As we can see, this method is based on the Δ method, however, it identifies not only the vortex region, but also the local strength and the local plane of swirling. It may be noted that $\Delta = 0$ and $\lambda_{ci} = 0$ are equivalent.

2.4. λ_2 method

The λ_2 method was proposed by Jeong and Hussain [11] and is popular due to its reliability in detecting vortices and the simplicity of the operations involved in the computation.