Applied Mathematical Modelling 40 (2016) 597–611

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Efficient numerical simulation method for evaluations of global sensitivity analysis with parameter uncertainty

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ARTICLE INFO

Article history:
Received 16 April 2013
Received in revised form 16 April 2015
Accepted 15 June 2015
Available online 3 July 2015

Keywords:
Epistemic and aleatory uncertainties
Failure probability
Improved importance sampling method
Variance-based importance measure

ABSTRACT

In this study, we propose an efficient numerical simulation method for structural systems with both epistemic and aleatory uncertainties to evaluate the effect of epistemic uncertainty on the failure probability measured by variance-based sensitivity analysis. The direct evaluation of this effect requires a "triple-loop" crude sampling procedure, which is time consuming. To circumvent the difficulty associated with the direct sampling-based procedure, we first construct an improved importance sampling (IS) method and an improved IS-based procedure is proposed for the efficient evaluation of the effect of epistemic uncertainty. The core of the proposed method is to construct the same IS probability density function for the failure probability corresponding to an individual realization of epistemic uncertainty. Using the proposed improved IS-based method, only one IS run with a set of input–output IS samples is needed to determine the estimated values of the effects for all epistemic uncertainties. Several examples are employed to demonstrate the feasibility of the proposed method for different situations. These examples demonstrate that the proposed method can obtain reasonably accurate results with fewer evaluations of the performance function compared with other existing methods.

1. Introduction

In practical engineering problems, the structural performance (or the output) always exhibits some degree of variation due to the presence of immanent uncertainties, such as the inaccuracy of the geometry, variability of material properties, fluctuations in external loads, and the errors that result from instrument measurements. These uncertainties are usually described by random variables, fuzzy variables, and uncertain-but-bounded variables [1]. For systems that are subject to these uncertainties, the main task of uncertainty analysis is to obtain the mathematical properties of the output, such as the probability density function (PDF), cumulative distribution function, statistical moments, and failure probability of the output for random uncertainties. For most practical applications, the output usually depends on many uncertain inputs. Thus, if each uncertain input is considered, the computational burden involved in the uncertainty analysis may be too great. In fact, each uncertain input generally has a unique impact on the output and the contribution of each uncertain input to the output may differ from each other. In addition, only the uncertainties that make high contributions will control the behaviors

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http://dx.doi.org/10.1016/j.apm.2015.06.009
0307-904X/

of the output. Thus, those with less impact can be fixed to their nominal values to decrease the dimensionality and complexity of the system under study.

Sensitivity analysis (SA) is often employed to identify the contribution of the uncertain input variables to the output, including local SA and global SA [2]. Local SA aims to determine how a small variation in an uncertain variable around a reference point changes the value of the output, but the main drawback of this approach is the dependence on the choice of the reference point. Global SA focuses on identifying the effect of each input uncertain variable on the output within the entire variation range of the variable. Global SA for random uncertainties has been investigated widely and a number of measures have been proposed, such as screening methods [3], scatter plots and correlation coefficients methods [4,5], linear regression methods [6], variance-based sensitivity methods [7,8], moment-independent methods [9–14], Kullback–Liebler divergence methods [15], information entropy-based methods [16], and non-parametric approaches [17,18]. Many global SA techniques were described in [6,19]. However, all of these indicators were proposed only for structural systems with epistemic input uncertainties. Previous studies [20–21] have also investigated another situation where the inputs of a model include aleatory uncertainties described by PDFs but the distribution parameters of the inputs are not known precisely and they are defined by epistemic uncertainties. Based on this idea, we proposed variance-based sensitivity measures of failure probability in the presence of epistemic and aleatory uncertainties [22]. In addition, we employed a Kriging-based method to estimate the proposed important measures for epistemic uncertainties. However, after conducting a detailed investigation of this method, we found that some issues still need to be addressed. First, the Kriging-based method is still a "double loop" sampling procedure and the computational cost may still be prohibitive. Second, as a surrogate technique, the Kriging-based method may depend on the type of the limit state function. Therefore, accuracy of the results is determined by the precision of the Kriging interpolation. Thus, in the present study, we propose an efficient numerical simulation method to overcome the difficulty associated with the Kriging-based method. The basic idea is to construct the same importance sampling (IS) PDF for the failure probability corresponding to each realization of the epistemic uncertainty. Based on the IS method, only one set of input–output IS samples generated from the IS PDF is needed to estimate all of the failure probabilities for the realizations of the epistemic uncertainty, so the computational cost is decreased. Several examples are provided to demonstrate the efficiency and accuracy of the proposed method.

The remainder of this paper is organized as follows. In Section 2, we derive the global SA for epistemic uncertainties in the presence of aleatory and epistemic uncertainties. In Section 3, we describe the direct sampling-based procedure for evaluating the global SA. In Section 4, we analyze the disadvantages of the traditional IS method and we propose the improved IS method, as well as the improved IS-based method for the efficient evaluation of the global SA associated with epistemic uncertainties. In Section 5, we present a simple example with only one variable to demonstrate the proposed improved IS method, before utilizing a high-dimensional linear problem with a semi-analytical solution to verify the applicability of the proposed method to high-dimensional problems, as well as giving two further engineering examples. Finally, we give our conclusions in Section 6.

2. Global SA for epistemic uncertainty

For structural systems that are subject to both epistemic and aleatory uncertainties, the performance function can be defined by:

$$y = g(\boldsymbol{x}; \boldsymbol{\theta}), \tag{1}$$

where y is the output, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ are n independent inputs, and $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_p)$ are p independent distribution parameters, which are epistemic uncertainties described by random variables using PDFs $f_{\boldsymbol{\theta}}(\boldsymbol{\theta})$.

Detailed discussions of epistemic uncertainty can be found in [20,21]. Further discussion of an analysis that maintains the separation of aleatory and epistemic uncertainty can be found in [23,24]. The variables $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ in [22] were assumed to be independent normal inputs, where the mean vector and standard deviation vector were $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ and $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, \ldots, \sigma_n)$, respectively. They mainly investigated the global SA of $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ based on an assumption that the epistemic uncertainties $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ were distributed as typical normal distributions with given means and standard deviations. In the present study, based on the assumption in [22], we focus on studying the global SA of $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$. For the sake of convenience, $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_n)$ are regarded as independent normal variables, which can be described by the joint PDFs $f_{\boldsymbol{\mu}}(\boldsymbol{\mu}) = \prod_{i=1}^{n} f_{\mu_i}(\mu_i)$ with $f_{\mu_i}(\mu_i) \sim N(\bar{\mu}_i, \sigma_{\mu_i})$, where $\bar{\boldsymbol{\mu}} = (\bar{\mu}_1, \bar{\mu}_2, \ldots, \bar{\mu}_n)$ and $\boldsymbol{\sigma}_\mu = (\sigma_{\mu_1}, \sigma_{\mu_2}, \ldots, \sigma_{\mu_n})$ are the mean vector and standard deviation vector of epistemic uncertainties $\boldsymbol{\mu}$, respectively. For a certain value $\tilde{\boldsymbol{\mu}}$, the aleatory uncertainties associated with the inputs $\boldsymbol{x}$ can be described by $f_{\boldsymbol{x}}(\boldsymbol{x}|\tilde{\boldsymbol{\mu}}, \boldsymbol{\sigma}) = \prod_{i=1}^{n} f_{x_i}(x_i|\tilde{\mu}_i, \sigma_i)$.

The standard deviation vector $\boldsymbol{\sigma}$ of $\boldsymbol{x}$ is a constant vector, so a generic value $\tilde{\boldsymbol{\mu}}$ of $\boldsymbol{\mu}$ corresponds to a $f_x(\boldsymbol{x}|\tilde{\boldsymbol{\mu}}, \boldsymbol{\sigma})$, thereby yielding a unique value $P_f(\tilde{\boldsymbol{\mu}})$ for the failure probability. Thus, there is a mathematical mapping [22] between the failure probability and $\boldsymbol{\mu}$. The failure probability $P_f(\boldsymbol{\mu})$ for Eq. (1) can be defined by [25]:

$$P_f(\boldsymbol{\mu}) = \int_{\Omega} I_F(\boldsymbol{x}) f_{\boldsymbol{x}}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\sigma}) d\boldsymbol{x}, \tag{2}$$

where $f_{\boldsymbol{x}}(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\sigma})$ represents the joint PDF of $\boldsymbol{x}$ with mean vector $\boldsymbol{\mu}$ and standard deviation vector $\boldsymbol{\sigma}$; $F = \{\boldsymbol{x}|g(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\sigma}) \leqslant 0\}$ is the failure region; Ω denotes the whole region; $I_F(\boldsymbol{x})$, while $I_F(\boldsymbol{x}) = 1$ if $g(\boldsymbol{x}; \boldsymbol{\mu}, \boldsymbol{\sigma}) \leqslant 0$ and $I_F(\boldsymbol{x}) = 0$ otherwise, is the failure