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E-Bayesian estimation for the geometric model based on record statistics

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ABSTRACT

This paper focuses on using a new method (E-Bayesian estimation) for computing estimates of the parameter and reliability function of the geometric distribution. The estimates are derived on the basis of a conjugate prior for the scale parameter. Our computations are based on the balanced loss function, which contains the symmetric and asymmetric loss functions as special cases. The results have been specialized to upper record values. Examples using both real and simulated record values are used to illustrate the application of the results. The results may be of interest in a situation where only record values are stored.

Contents

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1. Introduction

Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with discrete probability mass function $p(x) := P(X = x)$ and let $X_{1:n} \leqslant \ldots \leqslant X_{n:n}$ be the order statistics of the first $n(\geqslant 1)$ observations. Define sequences of record times U_n and record values X_n as follows:

$$U_1 = 1, \quad U_{n+1} = \min\{j : j > U_n, X_j > X_{U_n}\}.$$

These statistics are of interest and are important in several real-life problems involving the weather, economics, and sports data. The statistical study of record values started with Chandler [1], and has now spread in different directions. For more details and applications of the record values, see Ahsanullah [2] and Arnold et al. [3].

Consider the one-parameter geometric (Geo(θ)) distribution with probability mass function

$$p(x;\theta) = \theta(1-\theta)^{x-1}, \quad x = 1,2,3,\ldots, \quad 0 < \theta < 1 \tag{1}$$

and the reliability function

$$R(t) = \bar{p}(t;\theta) = (1-\theta)^t, \quad t > 0. \tag{2}$$

This distribution has many useful applications in agricultural, electronic engineering, cybernetics, economics, and reliability studies. For example, the pesticide application method depends on three things: the nature and habits of the target pest, the characteristics of the target site, and the properties of the pesticide and the suitability of the application equipment.

The number of applications to analyze a given pest is dependent on the probability of achieving successful control with a given application. Another important method is the number of pesticide applications made by farmers. So, farmers must decide how many times to apply the chosen pesticide. These frequencies can be modeled by a geometric distribution. For more details, see [4–6].

Treated applications in reliability theory are proposed by Mann et al. [7]. Gabriel and Neumann [8] used this distribution (Geo(θ)) as a model in meteorological models of weather cycles and precipitation amounts. Daniels [9] has investigated the representation of a discrete distribution as a mixture of geometric distributions, and has applied this to a busy-period distribution in equilibrium queueing systems. Sandland [10] proposed a building society membership scheme and a length of tenure scheme as models for the truncated geometric distribution with support $0, 1, \ldots, n-1$. For more details, see [11]. Özonur et al. [5] introduced some goodness-of-fit tests for the geometric distribution. Doostparast and Ahmadi [12] obtained a Bayesian estimation and prediction for geometric parameter θ on based on records. Bayesian estimation of this distribution under an entropy loss function was obtained by Xiong et al. [13]. Bayesian estimation and non-Bayesian estimation with a balanced loss function (BLF) are discussed by Zellner [14].

This paper is based on record statistics values from a geometric distribution. E-Bayesian and Bayesian approaches have been used to obtain the estimators of the parameter, and some lifetime parameters such as the reliability and hazard functions. Bayes estimators are developed under a balanced squared error loss (BSEL) function in Section 2. The estimates are derived on the basis of a conjugate prior for the parameter and BSEL in Section 3. Properties of E-Bayesian estimation are obtained in Section 4. Comparisons and computations are made between the new method and the corresponding Bayes and maximum-likelihood estimators of the target parameters via Monte Carlo simulation in Section 5. Also, practical examples using real record values are given to illustrate the application of the results. Finally, conclusions are given in Section 6.

We shall use the following form of the BLF introduced by Ahmadi et al. [15]:

$$L^q_{\rho,\omega,\delta_0}(\Lambda(\alpha),\delta) = \omega q(\alpha)\rho(\delta_0,\delta) + (1-\omega)q(\alpha)\rho(\Lambda(\alpha),\delta), \tag{3}$$

where $q(.)$ is a suitable positive weight function and $\rho(\Lambda(\alpha),\delta)$ is an arbitrary loss function used when $\Lambda(\alpha)$ is estimated by δ. The parameter δ_0 is a chosen priori "target" estimator of $\Lambda(\alpha)$, obtained, for instance, from the criterion of maximum likelihood, least squares, or unbiasedness. They give a general Bayesian connection between the case of $\omega > 0$ and the case of $\omega = 0$, where $0 \leqslant \omega < 1$. Through the choice of $\rho(\Lambda(\alpha),\delta) = (\delta - \Lambda(\alpha))^2$ and $q(\alpha) = 1$, the BLF reduced to the BSEL function, used by Ahmadi et al. [15], has the form

$$L_{\omega,\delta_0}(\Lambda(\alpha),\delta) = \omega(\delta-\delta_0)^2 + (1-\omega)(\delta-\Lambda(\alpha))^2 \tag{4}$$

and the corresponding Bayes estimate of the function $\Lambda(\alpha)$ is given by

$$\delta_{\omega,\Lambda,\delta_0}(\mathbf{x}) = \omega\delta_0 + (1-\omega)E(\Lambda(\alpha)|\mathbf{x}). \tag{5}$$

2. Bayesian estimation

Suppose we observe n upper record values $X_{U_1} = x_1, X_{U_2} = x_2, \ldots, X_{U_n} = x_n$ from the Geo(θ) distribution with probability mass function given by (1). The likelihood function can be written as