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A new boundary-based morphological model

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ABSTRACT

Mathematical morphology addresses the problem of describing shapes in an *n*-dimensional space using the concepts of set theory. A series of standardized morphological operations are defined, and they are applied to the shapes to transform them using another shape called the structuring element.

In an industrial environment, the process of manufacturing a piece is based on the manipulation of a primitive object via contact with a tool that transforms the object progressively to obtain the desired design. The analogy with the morphological operation of erosion is obvious. Nevertheless, few references about the relation between the morphological operations and the process of design and manufacturing can be found. The non-deterministic nature of classic mathematical morphology makes it very difficult to adapt their basic operations to the dynamics of concepts such as the ordered trajectory.

A new geometric model is presented, inspired by the classic morphological paradigm, which can define objects and apply morphological operations that transform these objects. The model specializes in classic morphological operations, providing them with the determinism inherent in dynamic processes that require an order of application, as is the case for designing and manufacturing objects in professional computer-aided design and manufacturing (CAD/CAM) environments. The operators are boundary-based so that only the points in the frontier are handled. As a consequence, the process is more efficient and more suitable for use in CAD/CAM systems.

1. Introduction

The continuous evolution of industrial technology has led to an increase in the quality of manufactured products. Computer-aided design and manufacturing (CAD/CAM) systems are now fundamental elements of the industry and are evolving at the same time as technology. Nevertheless, some problems still remain partially unsolved. Included among such problems is the complex problem of machining a piece using a tool. Although it is a problem that has been examined from many points of view with good results, it is still a complex problem that requires a very good knowledge of the problem and the use of ad-hoc techniques in many cases. A more general and formal mathematical model would be desirable.

The problem of machining a piece can be defined as a process of cutting a piece of material using a tool that moves according to a specific trajectory. A straightforward analogy can be established between the machining process and the formal concept of morphological erosion. The machining process can be interpreted as a morphological operation in which the

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structuring element (the tool) touches the target object (the manufactured piece), following a given direction. The process can also be likened to the design of an object, establishing a similar analogy when speaking of design tools and objects. Overall, we propose a definition of a morphological model to support the processes of machining and designing, attempting to establish both a generic formal model and a practical set of methods to solve real problems.

A short dissertation about mathematical morphology is now mandatory. Morphology is the study of shape, and mathematical morphology (MM) is mostly related to the mathematical theory of describing shapes using sets. It was first stated in 1964 when scientists Georges Matheron and Jean Serra applied the fundamental ideas of Minkowsky and Hadwiger to their studies on quantification of characteristics of minerals [1]. Later, Jean Serra made a generalization of mathematical morphology in a theoretical framework based on complete lattices (full set of points arranged with upper bound, supremum, and lower bound, infimum). This generalization brought flexibility to the theory, which meant that it could be applied to a larger number of structures and fields of application [2].

MM is based on set theory, with some elements from topology, geometry and discrete mathematics. The sets represent shapes in an n-dimensional space. A series of standardized morphological operations are applied to these sets. These operations are based on geometric relationships between the points of the sets. The aim of the morphological operations is to transform a set of points (the target object) using another set of points (the structuring element). The most widespread practical examples of this type of process are the morphological image filters based on the basic morphological operators of erosion and dilation. Another example is the process of designing and manufacturing shapes in CAD/CAM environments as discussed before.

References about mathematical morphology are abundant in various productive sectors. A good review of these applications can be found in [3], where the following fields appear: navigation systems, industrial control, medicine and biology, physics, aeronautics, geoscience and remote sensing, real-time systems and restoration processes. Image processing is one of the main uses of mathematical morphology. In the work of Soille and Pesaresi [4], Ghosh and Deguchi [5], Salembier et al. [6] and Velasco-Forero and Angulo [7], recent techniques that apply mathematical morphology to image processing in several fields are detailed. However, few references about the relation between the morphological operations and industrial processes can be found. A model that closely relates the process of design and manufacture is the trajectory-based design model, which bases object design on defining trajectories that are covered by modeling tools that simulate the material removed from the piece [8], although the model does not address the problem from a morphological point of view. One of the first examples of morphological processing in industry is topological modeling of the manufacturing process, which linked industrial machining with the concept of morphological erosion [9].

Delving into the link between the process of material removal and the morphological operation of erosion, we can identify the tool with the structuring element and the manufactured piece with the object to be eroded. However, the non-deterministic nature of classic mathematical morphology makes it impossible to adapt their basic operations to the dynamics of concepts such as the ordered trajectory. The morphological operation is not based on temporary orders because their original ones act on continuous sets of points and produce new continuous sets of points as a result without establishing a path order on its elements. This order relationship is necessary when the morphological paradigm must deal with dynamic processes such as the trajectory process. In addition, the morphological operation always obtains complete results without being able to apply partial transformations to objects that are involved in the operation.

The process of machining is, in essence, a process based on the surfaces of the shapes because the surface of the tool touches the surface of the piece. This fact leads us to propose to only compute the boundaries of the shapes so the calculations will be simpler and faster. Our aim is to demonstrate that a boundary-based computation is as valid as the traditional morphological methods. Some other authors have proposed algorithms that implement boundary-based morphological operations. Ragnemalm [10] and Meijster et al. [11] present techniques that apply morphological operations based on analytical calculations of distance between the boundary points of objects. Van Vliet and Verwer present algorithms for the calculation of erosion, dilation, skeletonization and propagation of images based on the boundary of shapes [12], and Wilkinson and Meijer [13] demonstrate a technique to classify images of microbiological organisms through the application of morphological operations to the boundary pixels of the images. However, the application of these techniques to the field of design and manufacturing is still unexplored.

To explore new possibilities of mathematical morphology in industrial environments, we present a formal framework inspired by the classic morphological paradigm that formally defines objects from their boundary and applies morphological operations that transform these objects. The model provides a specialization of the classic morphological operations, giving them the determinism of dynamic processes that require an order of application. The proposal is inspired by the needs of the field of design and manufacturing in CAD/CAM environments, but the results may be applied to other fields.

In Section 2, the formal model of deterministic boundary-based mathematical morphology is presented, along with the definition of the objects, the structuring elements and the set of morphological operations. The generic trajectory-based operation is also detailed, as it is the basis of the specialization that gives determinism to the morphological operations. At the end of this section, the operators of trajectory-based erosion and dilation are defined, as are some morphological filters and some interesting results that support the validity of the boundary-based morphological operators. Finally, in Section 3, some conclusions and findings are presented.