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Nonlinear vibration of functionally graded fiber-reinforced composite laminated cylindrical shells in hygrothermal environments

Hui-Shen Shen [a,*], De-Qing Yang [b]

[a] School of Aeronautics and Astronautics, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China
[b] School of Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China

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ABSTRACT

The effect of hygrothermal conditions on the linear and nonlinear free flexural vibration of anisotropic shear deformable laminated cylindrical shells is investigated. The cylindrical shell is made of fiber-reinforced composites (FRCs) with the reinforcement being distributed either uniformly (UD) or functionally graded (FG) of piece-wise type along the thickness of the shells. The motion equations are based on a higher order shear deformation shell theory with a von Kármán-type of kinematic nonlinearity. The hygrothermal effects are also included, and the material properties of FRCs are estimated through a micromechanical model and are assumed to be temperature-dependent and moisture-dependent. The equations of motion are solved by a singular perturbation technique along with a two-step perturbation approach to determine the linear and nonlinear frequencies of the FRC laminated cylindrical shells. Detailed parametric studies are carried out to investigate effects of material property gradient, the temperature change, the degree of moisture concentration, shell geometric parameter, stacking sequence, as well as the end conditions on the vibration characteristics of FRC shells with polymer matrix. The results show that the temperature/moisture variation has a moderately effect on the natural frequencies of the FRC cylindrical shells, but only has a small effect on the nonlinear to linear frequency ratios of the same shell.

1. Introduction

In recent years, fiber-reinforced composite (FRC) laminated structures have been widely used in the aeronautical, mechanical, civil, petrochemical and other engineering industries. Their components are often exposed to high temperature as well as moisture. The effect of environments on the material properties of composites was studied by many researchers, for example, Strife and Prewo [1], Adams and Miller [2], Ishikawa [3], Bowles and Tompkins [4] and Boukhoulda et al. [5]. Their results show that higher temperature and moisture reduce the elastic moduli and degrade the strength of composites. As a consequence, a careful evaluation of the effects of environmental exposure is required to find the nature and extent of their deleterious effects upon performance.

Many studies have been made on the bending, buckling and vibration of FRC laminated flat plates [6–16] and curved panels [17–28] in hygrothermal environments. However, investigations in nonlinear analysis of FRC laminated cylindrical shells

* Corresponding author.
E-mail address: hsshen@mail.sjtu.edu.cn (H.-S. Shen).

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under hygrothermal environmental conditions are limited in number. It is noted that in the above studies [23–28] the results were only for shell panels not pure shells. As has been shown in [19,29], the solutions of cylindrical shell and shell panel are different. For pure cylindrical shells, a full nonlinear postbuckling analysis for thin and moderately thick cross-ply laminated cylindrical shells subjected to combined loading of axial compression and external pressure under hygrothermal conditions was performed by Shen [29,30].

Nonlinear flexural vibration behavior of FRC cylindrical shells has received considerable attention [31–34]. However, although in a well-known reference case there seems to be a reasonable agreement, there are unresolved discrepancies between the results obtained by different authors [33]. This is mainly due to the fact that the assumed displacement does not satisfy either governing equations or boundary conditions. To the best of the authors' knowledge, there is no literature covering nonlinear flexural vibration behavior of FRC cylindrical shells in hygrothermal environments. In the aforementioned studies, however, the fiber reinforcements are usually assumed to be distributed uniformly in each ply and the fiber volume fraction does not vary spatially at the macroscopic level.

Functionally graded materials (FGMs) are a new generation of composite materials in which the microstructural details are spatially varied through nonuniform distribution of the reinforcement phase. Two kinds of FGMs are designed to improve mechanical behavior of plate/shell structures. One is functionally graded unidirectional fibers reinforced composites [35–37]. Another one, like functionally graded ceramic–metal materials, is functionally graded particles reinforced composites [38]. The concept of functionally graded material (FGM) can be utilized for the laminates by non-homogeneous distribution of fiber reinforcements into the matrix with a specific gradient so that the mechanical behavior of laminated shells can be improved. Unlike the functionally graded ceramic–metal materials, the material properties of functionally graded fiber reinforced composites (FG-FRCs) may be dependent of both temperature and moisture.

The nonlinear vibration characteristics of anisotropic laminated cylindrical shells were recently studied by Shen [39] where FRCs with metal matrix were considered. Unlike the FRCs with metal matrix, the material properties of FRCs with polymer matrix may be temperature-dependent and moisture-dependent. The present study extends the previous work [39] to the case of nonlinear flexural vibration of FRC laminated cylindrical shells with polymer matrix in hygrothermal environments. Two kinds of FRC laminated cylindrical shells, namely, uniformly distributed and functionally graded reinforcements, are considered. The shell is fully saturated such that the variation of temperature and moisture are independent of time and position. The motion equations are based on a higher order shear deformation theory with a von Kármán-type of kinematic nonlinearity and including the extension–twist, extension–flexural and flexural–twist couplings. The hygrothermal effects are also included, and the material properties of FRCs are estimated through a micromechanical model and are assumed to be temperature-dependent and moisture-dependent. A singular perturbation technique along with a two-step perturbation approach is employed to determine the linear and nonlinear frequencies of FRC laminated cylindrical shells. The numerical illustrations show the nonlinear vibration characteristics of FRC laminated cylindrical shells under different sets of hygrothermal environmental conditions.

2. Theoretical development

Some of the basic equations from Shen [39] are repeated here for clarity, with some changes included that are needed to make them applicable to FRC laminated cylindrical shells with polymer matrix in hygrothermal environments instead of FRC laminated cylindrical shells with metal matrix in thermal environments.

Consider a FRC cylindrical shell which consists of N plies of any kind. Each ply may have different value of fiber volume fraction, and the fiber reinforcement volume fraction distribution is functionally graded of piece-wise type in the thickness direction, as reported in Feldman and Aboudi [36]. The shell has mean radius R, length L and thickness h, of the shell, as shown in Fig. 1. The shell is exposed to moisture exposure and elevated temperature and is subjected to a transverse dynamic load $q(X,Y,\bar{t})$. The shell is referred to a coordinate system (X,Y,Z) in which X and Y are in the axial and circumferential directions of the shell and Z is in the direction of the inward normal to the middle surface, the corresponding displacements are designated by $\bar{U}$, $\bar{V}$ and $\bar{W}$. $\bar{\Psi}_x$ and $\bar{\Psi}_y$ are the rotations of normals to the middle surface with respect to the Y- and X-axes, respectively. The origin of the coordinate system is located at the end of the shell on the middle plane.

Based on Reddy's higher order shear deformation theory [40] with a von Kármán-type of kinematic nonlinearity and including hygrothermal effects, the motion equations for functionally graded FRC laminated cylindrical shells can be derived in terms of a transverse displacement $\bar{W}$, two rotations $\bar{\Psi}_x$ and $\bar{\Psi}_y$, and a stress function $\bar{F}$ as defined by $\bar{N}_x = \bar{F}_{,YY}$, $\bar{N}_y = \bar{F}_{,XX}$ and $\bar{N}_{xy} = -\bar{F}_{,XY}$, where a comma denotes partial differentiation with respect to the corresponding coordinates. These equations can be expressed by

$$\tilde{L}_{11}(\bar{W}) - \tilde{L}_{12}(\overline{\Psi}_x) - \tilde{L}_{13}(\overline{\Psi}_y) + \tilde{L}_{14}(\bar{F}) - \tilde{L}_{15}(\bar{N}^H) - \tilde{L}_{16}(\bar{M}^H) - \frac{1}{R}\bar{F}_{,XX} = \tilde{L}(\bar{W},\bar{F}) + \tilde{L}_{17}\left(\ddot{\bar{W}}\right) - \left(\tilde{I}_5\frac{\partial\ddot{\bar{\Psi}}_x}{\partial X} + \tilde{I}'_5\frac{\partial\ddot{\bar{\Psi}}_y}{\partial Y}\right) + q, \tag{1}$$

$$\tilde{L}_{21}(\bar{F}) + \tilde{L}_{22}(\bar{\Psi}_x) + \tilde{L}_{23}(\bar{\Psi}_y) - \tilde{L}_{24}(\bar{W}) - \tilde{L}_{25}(\bar{N}^H) + \frac{1}{R}\bar{W}_{,XX} = -\frac{1}{2}\tilde{L}(\bar{W},\bar{W}), \tag{2}$$