Applied Mathematical Modelling 39 (2015) 1526–1535

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Global exponential stability of periodic solutions for impulsive Cohen–Grossberg neural networks with delays

Huiyan Kang [a,b,*], Xinchu Fu [a], Zhenlong Sun [a]

[a] Department of Mathematics, Shanghai University, Shanghai 200444, PR China
[b] School of Mathematics and Physics, Changzhou University, Changzhou 213016, Jiangsu, PR China

ARTICLE INFO

Article history:
Received 20 November 2012
Received in revised form 1 March 2014
Accepted 16 September 2014
Available online 28 September 2014

Keywords:
Cohen–Grossberg neural network
Impulsive
Periodic solution
Global exponential stability

ABSTRACT

By constructing appropriate Lyapunov functions and using some inequality techniques and a fixed point theorem, some sufficient conditions are obtained to ensure the existence and global exponential stability of periodic solutions for impulsive Cohen–Grossberg neural networks with delays. The boundedness of the activation functions is not assumed. The criteria given can be easily verified and possess many adjustable parameters, which provide flexibility for the design and analysis of the system. Several previous results are improved and two examples are given to demonstrate the effectiveness of the theoretical results.

1. Introduction

The Cohen–Grossberg neural network models proposed by Cohen and Grossberg [1] have been widely applied to various problems arising in scientific and engineering fields, such as neural biology, signal and image processing, classification of patterns and quadratic optimization. In implementation of networks, time delays are inevitably encountered because of the finite switching speed of amplifiers. Moreover, many physical systems also undergo abrupt changes at certain moments due to instantaneous perturbations which lead to impulsive effects. So, the study of impulsive Cohen–Grossberg neural networks with delays is known to be an important problem in theory and applications. Many results on the existence, uniqueness, stability of solutions for this class of networks have been obtained in recent years (see [2–20]).

Sometimes neural networks present periodic oscillations so that their appropriate models are differential equations with periodic properties. Chen [8] and Bai [9] investigate stability of periodic solutions for impulsive neural networks with delays, but activation functions in [8] are required to be bounded, while delays in [9] are required to be discrete, and they both restrict the range on degree of jumps, which are also demanded in many recent articles. To the best of our knowledge, few authors have considered the stability of periodic solutions for impulsive Cohen–Grossberg neural networks with time-varying delays, which we will study in details in this paper.

We consider the following Cohen–Grossberg neural networks

* Corresponding author at: Department of Mathematics, Shanghai University, Shanghai 200444, PR China.
E-mail address: kanghuiyan@163.com (H. Kang).

http://dx.doi.org/10.1016/j.apm.2014.09.015
0307-904X/

$$\begin{cases} \dot{x}_i(t) = -c_i(x_i(t))[a_i(t,x_i(t)) - \sum_{j=1}^{n} b_{ij}(t) f_j(x_j(t)) \\ -\sum_{j=1}^{n} d_{ij}(t) f_j(x_j(t-\tau_{ij}(t))) - I_i(t)], \quad t \neq t_k, \\ \Delta x_{ik} = x_i(t_k^+) - x_i(t_k^-) = \gamma_{ik} x_i(t_k^-), \quad i = 1,\ldots,n, k \in N, \end{cases} \tag{1.1}$$

where $x_i(t)$ corresponds to the state of the ith unit at time $t, n \geq 2$ is the number of units in the networks. $c_i(u)$ represents an amplification function, $a_i(t,u)$ is an appropriately behaved function, $f_j(u)$ is the activation function, coefficients $b_{ij}(t)$ and $d_{ij}(t)$ denote the strength of the synaptic connection between the jth unit and the ith unit. $\tau_{ij}(t)$ corresponds to the finite speed of the axonal signal transmission and satisfies $0 \leqslant \tau_{ij}(t) \leqslant \tau$ (here τ is a constant), $I_i(t)$ is the external bias on the ith neuron at time t.

Throughout this paper, we assume $a_i(t,u), b_{ij}(t), d_{ij}(t), I_i(t)$ and $\tau_{ij}(t)$ are all continuous and T-periodic functions, and the sequence of positive real numbers $\{t_k\}_{k\in N}$ are strictly increasing, and $\lim_{k\to\infty} t_k = +\infty$. Moreover, we assume that there exists $q \in N$ such that

$$\{t_1,\ldots,t_q\} \subset (0,T),\ t_{k+q} = t_k + T,\ \gamma_{i(k+q)} = \gamma_{ik},\ k \in N, \quad i = 1,\ldots,n.$$

A solution $x(t) = (x_1(t),\ldots,x_n(t))^T$ of (1.1) on $[0,+\infty)$ is a piecewise continuous vector function which belongs to the space $PC([0,+\infty);R^n) = \{\phi : [0,+\infty) \to R^n : \phi(t)$ is continuous for $t \neq t_k, \phi(t_k^+), \phi(t_k^-) \in R^n$ and $\phi(t_k^-) = \phi(t_k)\}$. A function $\phi(t) \in PC([0,+\infty);R^n)$ is said to be T-periodic if $\phi(t+T) = \phi(t), t \geqslant 0$. The system (1.1) is supplemented with initial values given by

$$x(s) = \phi(s), \quad s \in [-\tau, 0],\ \phi \in PC([-\tau,0];R^n), \quad i = 1,\ldots,n.$$

And for convenience, we define $\|\phi\| = \sup_{-\tau\leqslant s\leq 0}(\sum_{i=1}^{n}|\phi_i(s)|^r)^{1/r}, \|x(t)\| = (\sum_{i=1}^{n}|x_i(t)|^r)^{1/r}, \phi \in PC([-\tau,0];R^n), x(t) \in PC([0,+\infty);R^n), r > 1$.

In addition, we formulate the following assumptions:

(H1) Each function $c_i(u)$ is bounded, positive, i.e., there exist constants $\underline{c}_i, \overline{c}_i$ such that $0 < \underline{c}_i \leqslant c_i(u) \leqslant \overline{c}_i,\ i = 1,\ldots,n$, and $\underline{c} = \min_{1\leqslant i\leqslant n}\{\underline{c}_i\}, \overline{c} = \max_{1\leqslant i\leqslant n}\{\overline{c}_i\}$;
(H2) $\frac{a_i(t,u)-a_i(t,v)}{u-v} \geqslant \mu_i(t) > 0$ for $u, v \in R, \mu_i(t)$ are continuous and T-periodic functions;
(H3) $f_i(\cdot)$ is Lipschitzian with Lipschitz constant $L_i > 0$, i.e.,

$$|f_i(u) - f_i(v)| \leqslant L_i|u - v|, \text{ for } u, v \in R, \quad i = 1,\ldots,n.$$

The remainder of this paper is organized as follows. In Section 2, we introduce some basic definitions and lemmas. The main results and some corollaries are given in Section 3. In Section 4, two examples are given to show the effectiveness of the proposed method. Finally, Section 5 concludes the paper.

2. Preliminaries

First, we present two definitions:

Definition 2.1. The solution $x^*(t,\phi^*)$ of the system (1.1) is said to be globally exponentially stable if there exist constants $M \geq 1$ and $\varepsilon > 0$ such that an arbitrary solution $x(t,\phi)$ of the system (1.1) satisfies the inequality

$$\|x(t,\phi) - x^*(t,\phi^*)\| \leqslant M\|\phi - \phi^*\|e^{-\varepsilon t},$$

for all $t \geq 0$.

Definition 2.2. Let $f(t) : R \to R$ be a continuous function, then the upper right derivative of $f(t)$ is defined as

$$D^+f(t) = \lim_{h\to 0^+} \sup \frac{f(t+h) - f(t)}{h}.$$

We quote two lemmas from [20,21] below:

Lemma 2.1 (Yang inequality [20]). *For $a \geq 0, b > 0$, the following inequality holds:*

$$ab \leqslant \frac{1}{p}a^p + \frac{1}{q}b^q,$$

where $p > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$.