Applied Mathematical Modelling 39 (2015) 1650–1664

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Upper and lower bounds for the optimal values of the interval bilevel linear programming problem

H. Mishmast Nehi *, F. Hamidi

Faculty of Mathematics, University of Sistan and Baluchestan, Zahedan, Iran

ARTICLE INFO

Article history:
Received 20 June 2012
Received in revised form 6 April 2014
Accepted 16 September 2014
Available online 5 October 2014

Keywords:
Bilevel programming
Interval coefficients
Hierarchical optimization

ABSTRACT

In this paper, we will investigate the interval bilevel linear programming (IBLP) problem. Recently, Calvete et al. have proposed two algorithms to find the worst and the best (upper and lower bounds) optimal values of the leader objective function in the bilevel linear programming (BLP) problem when the coefficients of the leader and the follower objective functions are interval. Through some examples, we will first show that the algorithm to find the worst optimal value of the leader objective function does not always yield a correct solution. Then, after investigating its drawbacks, we will propose a revised algorithm with which the previous examples will yield correct solutions. Finally, it will be extended to the general BLP problem wherein all the coefficients are interval. It is, of course, possible to easily find the upper and the lower bounds for the lower level objective function too.

1. Introduction

The bilevel programming (BP) model has been presented for a decision making process that consists of two decision makers in a hierarchical structure. In fact, BP is a model for a static two person game (the leader player in the upper level and the follower player in the lower level) wherein each player tries to optimize his/her personal objective function under dependent constraints; this game is sequential and non-cooperative [1]. The decision making variables are divided between the two players and one's choice affects the other's benefit and choices. In other words, BP consists of two nested optimization problems where the constraint region of the upper level problem is implicitly determined by the lower level problem. A major characteristic of the bilevel linear programming (BLP) problem is its non-convexity that makes it an NP-hard problem [2].

In real cases, the coefficients of an optimization problem may not be precise, i.e. they may be interval, fuzzy or random; same is the case with BLP problems. The papers about BLP problems with interval coefficients are not many. Sometime ago, we proposed an algorithm for the solution of the fuzzy BLP problem wherein use has been made of the solution of the BLP problem with interval coefficients [3]. In [3], using "α-cut", an interval BLP is obtained from a fuzzy one. The best and the worst optimal values of the objective function will be found first, and then a linear piecewise trapezoidal approximate fuzzy number will be presented for the optimal value of the leader objective function related to the fuzzy BLP problem. Calvete and Galé [4] have worked on the BLP problem when the coefficients of the leader and follower objective functions are interval and have found a range for the optimal value of the leader objective function. In fact, they have calculated the best and the worst optimal values of the leader objective function by their proposed algorithms. When only the coefficients of the leader objective function are interval, we can find the best and the worst optimal values of the leader objective function by solving

* Corresponding author.
E-mail addresses: hmnehi@hamoon.usb.ac.ir (H. Mishmast Nehi), f_hamidi@math.usb.ac.ir (F. Hamidi).

http://dx.doi.org/10.1016/j.apm.2014.09.021
0307-904X/

two BLP problems, but when the coefficients of both the leader and follower objective functions are interval, finding these two values becomes complicated because in such a case, the leader feasible region can change with changes in the coefficients of the follower objective function. Ren et al. [5] have studied on the bilevel linear programming problems with fuzzy random variable coefficients in objective functions. They have applied an interval programming approach based on the "α-cut" to obtain the best and worst stochastic models. Then using expectation optimization model, the best and worst stochastic problems are transformed into the deterministic problems. Li and Fang [6], have presented an efficient genetic algorithm for interval BLP Problems.

In this paper, we will first investigate Calvete et al.'s proposed algorithms and show that the one which determines the worst optimal value of the leader objective function does not always yield a correct solution. Then, revising it, we will implement it on some examples and, finally, extend it to the general BLP problem with fully interval coefficients. To continue, the paper organization is as follows: a short introduction of the BLP problem in Section 2; a brief study of Calvete et al.'s paper in Section 3; presentation of a revised algorithm for the determination of the worst optimal value of the leader objective function in Section 4; and finally, determination of the generalized algorithms for the IBLP problems in Section 5.

2. Bilevel linear programming

Here, we will review the definitions, properties and theorems of the BLP problem. In the BP problem when the objective functions and constraints are linear, we will have the following BLP problem:

$$\begin{aligned} &\min_{\mathbf{x}\in X} \quad F(\mathbf{x},\mathbf{y}) = \mathbf{cx} + \mathbf{dy} \\ &s.t. \quad A^1\mathbf{x} + B^1\mathbf{y} \geqslant \mathbf{b}^1 \\ &\min_{\mathbf{y}\in Y} \quad f(\mathbf{x},\mathbf{y}) = \mathbf{ay} \\ &s.t. \quad A^2\mathbf{x} + B^2\mathbf{y} \geqslant \mathbf{b}^2, \end{aligned} \tag{1}$$

where $\mathbf{x} \in X \subset R^{n_1}$; $\mathbf{y} \in Y \subset R^{n_2}$; $F,\ f : X \times Y \longrightarrow R$; $\mathbf{c} \in R^{n_1}$; $\mathbf{d},\ \mathbf{a} \in R^{n_2}$; $\mathbf{b}^1 \in R^{m_1}$; $\mathbf{b}^2 \in R^{m_2}$; $A^1 \in R^{m_1\times n_1}$; $B^1 \in R^{m_1\times n_2}$; $A^2 \in R^{m_2\times n_1}$; $B^2 \in R^{m_2\times n_2}$. Following are the definitions for the BLP problem (1).

Definition 2.1 [7].

(a) Constraint region of the BLP:

$$S = \{(\mathbf{x},\mathbf{y}) : \mathbf{x} \in X, \mathbf{y} \in Y, A^1\mathbf{x} + B^1\mathbf{y} \geqslant \mathbf{b}^1, A^2\mathbf{x} + B^2\mathbf{y} \geqslant \mathbf{b}^2\}.$$

(b) Projection of S onto the leader's decision space:

$$S_1 = \{\mathbf{x} \in X : \exists \mathbf{y} \in Y, A^1\mathbf{x} + B^1\mathbf{y} \geqslant \mathbf{b}^1, A^2\mathbf{x} + B^2\mathbf{y} \geqslant \mathbf{b}^2\}.$$

(c) Feasible set for follower for each fixed $\mathbf{x} \in X$:

$$S(\mathbf{x}) = \{\mathbf{y} \in Y : B^2\mathbf{y} \geqslant \mathbf{b}^2 - A^2\mathbf{x}\}.$$

(d) Follower's rational reaction set for $\mathbf{x} \in S_1$:

$$M(\mathbf{x}) = \{\mathbf{y} \in Y : \mathbf{y} \in \operatorname{argmin}\{f(\mathbf{x},\hat{\mathbf{y}}) : \hat{\mathbf{y}} \in S(\mathbf{x})\}\}.$$

(e) Inducible region:

$$FR = \{(\mathbf{x},\mathbf{y}) : \mathbf{x} \in S_1, \mathbf{y} \in M(\mathbf{x})\}.$$

X and Y are usually considered as follows:

$$X = \{\mathbf{x} \in R^{n_1} : \mathbf{x} \geqslant \mathbf{0}\}, \quad Y = \{\mathbf{y} \in R^{n_2} : \mathbf{y} \geqslant \mathbf{0}\}.$$

Suppose $S \neq \emptyset$ and it is compact and $M(\mathbf{x})$ is a point-to-point map. One important feature of the BLP is that FR is formed by faces of S (Theorem 1 in [8]) and another is that the BLP optimal solution occurs at a vertex of S (Theorem 2 in [8]). Different solution methods, based on vertex enumeration, penalty function, KKT conditions and so on, have been presented for this problem [9–13].

3. BLP problem with interval coefficients in the objective functions (BLPIC)

In this section, first we will briefly review the definitions, theorems and algorithms presented in [4] for the solution of the BLP problem with interval coefficients in the objective functions. Then, giving some examples, we will show that there are some drawbacks in the KBW algorithm for the determination of the worst optimal value of the leader objective function. It is to be noted that in the BLP model studied in Calvete et al.'s paper, it is assumed that the upper level constraints do not exist.

Here, the preliminaries related to the BLPIC problem will be reviewed.