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Wave propagation at the boundary surface of an elastic and thermoelastic diffusion media with fractional order derivative

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ABSTRACT

The purpose of this research is to study the reflection and refraction of obliquely incident plane wave at the interface of elastic and thermoelastic diffusion media with fractional order derivative. Lord–Shulman [1] theory of thermoelasticity using the methodology of fractional calculus is used to investigate the problem. It is noticed that three type of longitudinal waves and one transverse wave propagate in an isotropic thermoelastic diffusion medium. The amplitude and energy ratios of various reflected and refracted waves are obtained and it is found that these are the functions of incidence angle and frequency of that incident wave. The variation of modulus of amplitude ratios and energy ratios with the angle of incidence for different fractional orders are computed numerically using continuous boundary conditions for the copper material. The computed numerical results are depicted graphically for the variation of energy ratios with the angle of incidence. It has been verified that the sum of energy ratios is equal to unity at the interface. Some particular cases of interest are also deduced from the present investigation.

1. Introduction

During recent years, many fascinating models are developed by using fractional calculus to study the physical processes significantly within the area of heat conductivity, diffusion, viscoelasticity, mechanics of solids, control theory, electricity, etc. it is been accomplished that the utilization of fractional order derivatives and integrals results in the formulation of certain physical problems that is more economical and useful than the classical approach. There exists several material and physical situations like amorphous media, colloids, glassy and porous materials, biological materials/polymers, transient loading, etc. wherever the classical thermoelasticity based on Fourier type heat conduction breaks down. In such cases, one has to use a generalized thermoelasticity theory based on an anomalous heat conduction model involving time fractional (non integer order) derivatives. The importance of the fractional order theory is because of their non-local property i.e. present state of the system does not depend solely on its present state however additionally on all its historical states.

The first application of fractional derivatives was given by Abel [2] who applied fractional calculus within the solution of an integral equation that arises within the formulation of the tautochrone problem. Caputo [3] gave the definition of fractional derivatives of order $\alpha \in (0,1]$ of absolutely continuous function. Caputo and Mainardi [4,5] and Caputo [6] found good agreement with experimental results on using fractional derivatives for description of viscoelastic materials and established

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the association between fractional derivatives and the theory of linear viscoelasticity. Oldham and Spanier [7] studied the fractional calculus and proved the generalization of the thought of derivative and integral to a non-integer order.

Rossikhin and Shitikova [8] has given applications of fractional calculus to numerous issues of mechanics of solids. Povstenko [9] proposed a quasi-static uncoupled theory of thermoelasticity based on the heat conductivity equation with a time-fractional derivative of order α. As a result of the heat conductivity equation within the case $1 \leqslant \alpha \leqslant 2$ interpolates the parabolic equation $\alpha = 1$ and also the wave equation $\alpha = 2$, this theory interpolates a classical thermoelasticity and a thermoelasticity without energy dissipation. He additionally obtained the stresses corresponding to the fundamental solutions of a cauchy problem for the fractional heat conductivity equation for one-dimensional and two-dimensional cases.

Podstrigach [10] considered the problem of thermodiffusion in classical elastic material and investigated the elemental corollaries and differential equations. Podstrigach and Pavlina [11] constructed the differential equations of thermodynamical processes in an n-component solid solution. Podstrigach [12] presented the diffusion theory of strain of an isotropic solid medium. Podstrigach [13] examined the diffusion theory of inelasticity of metals. Nowacki [14–17] developed the theory of thermoelastic diffusion by using coupled thermoelastic model.

Povstenko [18] investigated the nonlocal generalizations of the Fourier law and heat conduction by using time and space fractional derivatives. Youssef [19] proposed a new model of thermoelasticity theory in the context of a new consideration of heat conduction with fractional order and proved the uniqueness theorem. Povstenko [20] investigated the fractional radial heat conduction in an infinite medium with a cylindrical cavity and associated thermal stresses.

Ezzat [21] studied the problem of state space approach to thermoelectrical fluid with fractional order heat transfer to solve a one-dimensional application for a conducting half space of thermoelectric elastic material. Povstenko [22] investigated the generalized Cattaneo-type equations with time fractional derivatives and formulated the theory of thermal stresses. Plane waves in anisotropic thermoelastic diffusion media with fractional order derivative was given by Kumar and Gupta [23]. Fractional order GN model on thermoelastic interaction in an anisotropic plate was introduced by Abbas [24].

Kumar and Kumar [25] studied the reflection and refraction of incident waves at the boundary surface of an elastic half-space and initially stressed thermoelastic with voids half-space. Authors like Sinha and Elsibai [26], Kumar and Sarthi [27] investigated the problems of reflection and refraction at the boundary surface by considering different mathematical model.

In the present paper, the reflection and refraction at a plane interface between an elastic solid half-space and a thermoelastic diffusion solid half-space with fractional order derivative has been analyzed. Using potential functions, the amplitude ratios of various reflected and refracted waves to that of incident wave are derived and are further used to find the expressions of energy ratios of various reflected and refracted waves to that of incident wave. These energy ratios are presented graphically and comparison is given for different fractional orders. The law of conservation of energy at the interface is verified.

2. Governing equations

Following Ezzat and Fayik [28], the governing equations for homogeneous isotropic generalized thermoelastic diffusion with fractional order derivative, in the absence of body forces and heat sources, are the constitutive relations

$$\sigma_{ij} = 2\mu e_{ij} + \delta_{ij}[\lambda e_{kk} - \beta_1 T - \beta_2 C], \tag{1}$$

$$\rho T_0 S = \rho C_E T + \beta_1 T_0 e_{kk} + a T_0 C, \tag{2}$$

$$P = -\beta_2 e_{kk} + bC - aT. \tag{3}$$

The equations of motion

$$(\lambda + 2\mu)\nabla(\nabla.\mathbf{u}) - \mu(\nabla \times \nabla \times \mathbf{u}) - \beta_1 \nabla T - \beta_2 \nabla C = \rho \ddot{\mathbf{u}}. \tag{4}$$

The heat conduction equations

$$K\nabla^2 T = \left(1 + (\tau_0^\alpha/\alpha!)(\partial^\alpha/\partial t^\alpha)\right)\left[\rho C_E \dot{T} + \beta_1 T_0(\nabla.\dot{\mathbf{u}}) + a T_0 \dot{C}\right]. \tag{5}$$

The equation of mass diffusion

$$D\beta_2 \nabla^2(\nabla.\mathbf{u}) + Da\nabla^2 T + \left(1 + \left((\tau^0)^\alpha/\alpha!\right)(\partial^\alpha/\partial t^\alpha)\right)\dot{C} - Db\nabla^2 C = 0, \tag{6}$$

where λ, μ are the Lame's constants, ρ is the density assumed to be independent of time, u is displacement vector, α denotes the fractional order parameter, K is the coefficient of thermal conductivity, C_E is the specific heat at constant strain, τ^0 is the diffusion relaxation times and τ_0 is thermal relaxation time, $T = \Theta - T_0$ is small temperature increment, Θ is the absolute temperature of the medium; T_0 is the reference temperature of the body chose such that $|(T/T_0)| \ll 1$, C as the concentration of the diffusive material in the elastic body, a, b and D are respectively, the coefficients describing the measure of thermo diffusion, mass diffusion effects and thermoelastic diffusion constant respectively, σ_{ij}, e_{ij} are the components of the stress and strain respectively, e_{kk} is the dilatation, $\beta_1 = (3\lambda + 2\mu)v_t$ and $\beta_2 = (3\lambda + 2\mu)v_c$, v_t is the coefficient of thermal linear expansion, v_c is the coefficient of linear diffusion expansion. Here $\alpha = \tau_0 = \tau^0 = 0$ for coupled theory (CT) and $\alpha = 1$ for Lord–Shulman (LS) theory of thermoelasticity.