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Motion equation of nonholonomic wheeled mobile robotic manipulator with revolute–prismatic joints using recursive Gibbs–Appell formulation

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ABSTRACT

The main intent of this paper is to represent a symbolic algorithm, capable of deriving the equations of motion of N-rigid link manipulators with revolute–prismatic (R–P) joints, which mounted on a mobile platform. The presence of prismatic joints besides the revolute ones, as well as the nonholonomic characteristics of the mobile platform makes the derivation of governing equations difficult. So, to derive the kinematic and dynamic equations of motion of such a complex system, and also to avoid computing the Lagrange multipliers associated with the nonholonomic constraints, the application of recursive Gibbs–Appell (G–A) formulation is applied. For modeling the system completely and precisely, the dynamic interactions between the manipulator and the mobile platform, the coupling effects due to the simultaneous rotating and sliding motion of the rigid arms, as well as both nonholonomic constraints associated with the no-slipping and the no-skidding conditions are included. Moreover, to improve the computational efficiency of the proposed systematic algorithm, all mathematical operations are done by only 3×3 and 3×1 matrices. Finally, a numerical simulation for a mobile manipulator with three R–P joints is performed, by using a developed computer program, to show the ability of this algorithm in deriving and solving the equations of motion of such systems.

1. Introduction

Most of the investigations on the dynamics of mobile robotic manipulators are confined to manipulators with revolute joints. Dynamics of rotating arms sliding in prismatic joints is an important problem in many engineering applications. Combining such a system with a mobile platform makes these robots be able to execute manipulation tasks in a much larger workspace than a fixed-base manipulator. In fact, the mobility characteristic of the mobile base with the dexterous manipulation abilities of the manipulator, makes these robots are progressively used in different applications such as rescue operations, military missions, planetary exploration, nuclear reactor maintenance, mine detection, welding, painting, agriculture and firefighting.

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Any mobile platform with two independent driving wheels has three constraints, two of them are nonholonomic and the third one is holonomic. In such a system, the base must be moved in the direction of its main axis of symmetry and cannot move instantly in any arbitrary direction. This non-integrable kinematic constraint is called a nonholonomic one. On the other hand, any mobile robot with three degrees of freedom of motion in the plane has become known as a holonomic one [1]. Because of the complexity arising from the modeling of nonholonomic constraints in wheeled mobile manipulators, most previous investigators considered only holonomic motion of the platform [2–4]. However, to fully exploit the benefits offered by a mobile manipulator, it is necessary to develop an explicit, complete and precise dynamic model for these kinds of robots. The effects of the dynamic interaction between the manipulator and the mobile platform have been studied by Liu and Lewis [5], Wiens [6], Meghdari et al. [7] and Yamamoto et al. [8]. Chen and Zalzala [9] presented an approach for the modeling and motion planning of a mobile manipulator system under the nonholonomic constraint by using Newton–Euler formulations. But all nonholonomic constraints executed in such systems are not considered in their models. Some investigators like Calbaugh [10] augmented these constraints with dynamic equations, while others like Yamamoto et al. [11,12] tried some sort of coordinate reduction after included all nonholonomic constraints that imposed on such systems. In their works augmenting the constraints with the system's equations of motion is done by using the Lagrange multipliers methodology.

The use of Lagrangian procedure involves Lagrange multipliers which are associated with nonholonomic constraints. With Lagrangian mechanics, these Lagrange multipliers cannot be eliminated from the formulation before deriving the equations of motion. On the other hand, after deriving the motion equations, eliminating the Lagrange multipliers is known to be a cumbersome and computationally inefficient task. So, to avoid Lagrange multipliers, Thanjavur and Rajagopala [13] modeled an autonomously guided vehicle, using Kane's equations. They mentioned the benefits of using Kane's methodology to model vehicles and exploited some of the tools to incorporate nonholonomy. Also, Tanner and Kyriakopouos [14] offered a comprehensive model for a mobile manipulator system, based on Kane's approach. In their work, a set of dynamic equations and a set of constraint equations for multiple mobile manipulators are presented. Another method for avoiding the Lagrange multipliers was proposed by Saha [15] and Angeles [16], in which the dynamics of a rolling robot with conventional wheels was studied. They used the concept of the Orthogonal Complement [17] of the matrix of nonholonomic constraints to develop the equations of motion.

The more arms mounted on the platform, the more laborious and complicated procedures required in the complete modeling of the wheeled mobile manipulators. This fact intensifies when the arms of the manipulator have a simultaneous revolute and reciprocate motion. So, to increase the efficiency of deriving the inverse and forward dynamic equations, using a recursive formulation is inevitable. In spite of the great number of proposed recursive algorithms for fixed-base manipulators [18–21], quite few efforts have been made on the systematic modeling of the mobile manipulators. In the field of mobile robotic manipulators, Yu and Chen [22] applied the principle of virtual work to derive the motion equations of the nonholonomic mobile manipulator systems, in recursive form. The proposed approach is considered as a general one for modeling the nonholonomic wheeled mobile manipulators. Recently, a recursive algorithm, which is based on Newton–Euler dynamics, is suggested by Boyer and Shaukat [23]. This algorithm performs calculation of the control torques, as well as the overall rigid motions involved in locomotion tasks. Also in Ref. [24], a recursive Newton–Euler formulation is applied to derive the inverse and forward dynamic of different robotics systems by Khalil. One of the cases that had been considered was a robotic manipulator with moving base.

The equations of motion of mobile robotic manipulators have been solved by a large variety of formulations that can be found in technical literatures, including the Newton–Euler equations by Li and Zhao [25]; Lagrange–d'Alembert formulation by Chung et al. [26]; Kane's equations by Meghdari et al. [27]; and Lagrange's equations by Seidi and Markazi [28]. Motion equations by G–A formulation have been used very seldom for deriving the dynamic equations of manipulator robots. In the field of robotics, more recent investigations can be found in [29] where G–A formulation is used for motion equations of snake-like robots by Vosoughi et al. [30,31] where inverse and forward dynamic equations of motion of N-rigid links are presented in recursive form by Mata et al. finally in [32] where motion equations of elastic link manipulators are presented by Korayem et al. The most relevant paper in this study belongs to Korayem et al. [33] in which the equations of motion of N-rigid links with only revolute joints that mounted on a mobile platform has been derived based on the G–A formulation.

As mentioned above, this paper focuses on the study of dynamic modeling of nonholonomic wheeled mobile robotic manipulators with R–P joints by using recursive G–A formulation. So, the rest of the paper is organized as follows: In Section 2 the kinematics of the system is described. Section 3 is devoted to obtain the inverse dynamic equations of motion including the Gibbs' function of the system and its derivatives. Forward dynamic equations of the system in recursive form are presented in Section 4. A computational simulation is performed in Section 5 to show the effectiveness of proposed method. In Section 6 the conclusions from the present work are summarized and the merits of the proposed method are highlighted. And finally in Appendix A, a recursive algorithm is proposed that generates the equations of motion systematically.

2. Kinematics of the system

This Section is composed of two parts. At first the kinematics of the manipulator will be presented. And then, the kinematics of the mobile platform and also the kinematics of the left and right driving wheels will be considered.