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Short communication

Spectral stochastic modeling of uncertainties in nonlinear diffusion problems of moisture transfer in wood

Miroslav Trcala

Department of Wood Science, Mendel University in Brno, Czech Republic

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ABSTRACT

This paper deals with the stochastic numerical analysis of moisture transfer in wood with the random diffusion coefficient after heating of wood (when temperature is already constant). The simulation is based on the unsteady-state nonlinear (the model respects the dependence of diffusion coefficients on moisture and constant temperature) diffusion of moisture with respect to the orthotropic nature of wood. The spectral solution of this problem is based on discretization the resulting random field (moisture) in the stochastic dimension by the orthogonal polynomials (generalized polynomial chaos algorithm). A Galerkin projection is applied in the stochastic dimension to obtain the deterministic set of partial differential equations that is solved by finite element method.

The main purpose of this paper is to demonstrate that the stochastic spectral method based on polynomial chaos expansion can be more efficient in modeling uncertainties associated with moisture transfer in wood than Monte Carlo method mainly when considering a small number of random variables. This spectral approach has a big advantage over the Monte Carlo method (statistical approach) in terms of computer time. Numerical example of diffusion of moisture in convective drying of wood is given and there is shown that the results (mean and the standard deviation) obtained with the stochastic spectral method are in good agreement with the results of the Monte Carlo simulations.

1. Introduction

Diffusion problems of moisture transfer in wood occur in many areas including wood drying, thermal treatment of wood, building physics, etc. In this work, the stochastic diffusion problem of wood drying is solved because there are some uncertainties in values of diffusion coefficients of wood. Wood drying is dynamic process where boundary conditions are different in time. These conditions are regulated in influence of require results of wood drying. The most important is then optimization of this process with respect on efficiency and economy of process and quality of dried material. The aim is to decrease the energy consumption (and also decrease drying time) and simultaneously increase the drying quality. These two requirements are contradictory and there is need to optimize the process. There is important the understanding of moisture and heat transfer in wood during drying. Up to present was designed many theoretical models for drying of wood [1–4].

The models of wood drying are multiphysics models based on analysis of coupled temperature, moisture and strain–stress fields in wood [5–8]. A good mathematical description of the physical problem in porous material was given in Luikov [9,10] and Whitaker [11]. They assumed that the moisture transfer is analogical to heat transfer and is dependent not only on moisture gradient [12] but also on temperature gradient.

Numerical simulations in diffusion problems rely on obtaining accurate physical properties of materials. In many situations, it is difficult to obtain the accurate values of material properties. Most of building envelopes are porous media and

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their structure is complex and material properties are difficult to determine. Thus, uncertainty quantification and propagation in physical systems appear as a critical path for the improvement of the prediction of their response [13]. In the last two decades, a growing interest has been devoted to a new family of methods, called spectral stochastic methods, for the propagation of uncertainties through physical models governed by stochastic partial differential equations. In this paper we focus on uncertainty propagation with the polynomial chaos (PC) method, which is based on the spectral representation of the uncertainty. The biggest advantage of the spectral PC method is given mainly by low computational costs.

Several methods have been used to model and propagate uncertainty in stochastic computational simulations [14]. Several researchers have studied and implemented the PC approach for a wide range of problems. Ghanem and Spanos [15] and Ghanem [16,17] applied the PC method to several problems of interest to the structures community. Mathelin et al. [18] studied uncertainty propagation for a turbulent, compressible nozzle flow by this technique. Xiu and Karniadakis [19] analyzed the flow past a circular cylinder and incompressible channel flow by the PC method and extended the method beyond the original formulation of Wiener [20] to include a variety of basis functions. Walters [21] applied the PC method to a two-dimensional steady-state heat conduction problem for representing geometric uncertainty. Following this effort, an implicit compact PC formulation was implemented for the stochastic Euler equations [22]. The PC method for the propagation of uncertainty in computational simulations involves the substitution of uncertain variables and parameters in the governing equations with the polynomial expansions.

In general, an intrusive approach will calculate the unknown polynomial coefficients by projecting the resulting equations onto basis functions (orthogonal polynomials) for different modes. As its name suggests, the intrusive approach requires the modification of the deterministic code and this may be inconvenient for many complex computational problems since the modification of the existing code can be difficult, expensive, and time consuming. To alleviate this inconvenience, non-intrusive methods have been investigated by many researchers. Most of the non-intrusive PC approaches in the literature are based on sampling [23–25] or quadrature methods [23,26] to determine the projected polynomial coefficients.

Before their introduction, the standard was to employ Monte Carlo (MC) methods to sample the inputs, compute the output for each sample, and aggregate statistics of the output [27]. In fact, this is still the most widely used method in practice due to its robustness and ease of implementation. Monte Carlo integration consists in choosing for the integration points K independent random samples (in practice pseudo-random samples) of the random input variables ξ. The estimation being random, a prediction is then given with a certain confidence interval. Standard deviation of the estimator equals $K^{-1/2}\sigma$, where σ is standard deviation of the resulting (output) random variable. The convergence rate of this estimator, in $O(K^{-1/2})$, is independent of the stochastic dimension, which makes possible the use of Monte Carlo technique in very high stochastic dimension [13].

However, the MC methods suffer from a dreadfully slow convergence rate for a smaller number of random input variables (in this work is only one random input variable – diffusion coefficient) comparing with the spectral methods. And if each sample evaluation is expensive, such as the solution of a partial differential equation, then obtaining hundreds (sometimes even thousands) of samples may be entirely infeasible. Thus, the initial applications of spectral methods showed orders-of-magnitude reduction in the work needed to estimate statistics with comparable accuracy [28]. Such results spurred interest in applying spectral methods to differential equations with stochastic (i.e. parameterized) inputs [29]. This paper presents a spectral polynomial chaos method for the propagation of input uncertainty in diffusion coefficient of wood.

1.1. Generalized polynomial chaos and Galerkin projection

An important aspect of spectral representation of uncertainty is that one may decompose a random function (or variable) into separable deterministic and stochastic components [14].

There are possible many different choices for basis functions depending on the type of the probability distribution selected for the input uncertainty (original polynomial chaos with Hermite polynomials was extended into generalized polynomial chaos) [28,30].

Using the generalized polynomial chaos we can represent second-order stochastic processes through the expansion:

$$\mathbf{u}(\mathbf{x}, t, \xi(\omega)) = \sum_{i=0}^{\infty} \mathbf{u}_i(\mathbf{x}, t)\Phi_i(\xi(\omega)), \tag{1}$$

where $\mathbf{x}$ is an element of the coordinate space, t denotes the time, ω is a point in the sample space, $\{\Phi_i(\xi(\omega))\}$ is random basis with orthogonal polynomials from the Askey-schemein terms of a random vector $\xi = \{\xi_i(\omega)\}_{j=1}^{N}$ and $\mathbf{u}_i(\mathbf{x}, t)$ is the deterministic component of the expansion. The combination of random vector and polynomials is carefully selected based on the distribution of the random input. This means that a set of orthogonal polynomials is created in such a way that they are orthogonal with respect to a weighting function which equals the probability density function of the random input [31]. This generalized polynomial chaos has later been further generalized for arbitrary random inputs [31,30,32].

Lets have this stochastic partial differential equation

$$L(\mathbf{x}, t, \xi; \mathbf{u}) = f(\mathbf{x}, t, \xi), \tag{2}$$

where $\mathbf{u} = \mathbf{u}(\mathbf{x}, t, \xi(\omega))$ is the solution and $f(\mathbf{x}, t, \xi)$ is the source term. L is a general differential operator (linear or nonlinear) that may contain spatial derivatives and time derivatives and assume

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