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Auxiliary model method for transfer function estimation from noisy input and output data

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ABSTRACT

In order to get the unbiased parameter estimation of transfer function from noisy input and output data, this paper presents an auxiliary model based method. The basic idea is to replace the unmeasurable outputs in the information vector of the system by auxiliary model outputs, and to modify the recursive least squares parameter estimation by the errors between the system output and auxiliary model output. The proposed method improves the precision of parameter estimates, and has wide application in modelling and control engineering without strict mathematical assumption. Finally, the advantages of the proposed approach are shown by simulation tests.

1. Introduction

As the development of information technology (IT), we are going to be in a complete new data technology (DT) era. System identification, which is one traditional research direction in the field of data-driven based mathematical modeling technology, which deals with problems of building mathematical model of a dynamical system based on observed data from the systems [1], has ushered in a new focus recently [2–4]. High precision requirements is a difficult problem for many engineering and technology. Unbiased parameter estimation is a technique to improve the accuracy of parameter estimates and occupies an important position in the field of system identification. In recent two or three decades, unbiased parameter estimates have been well studied for Autoregressive model with exogenous input (ARX) systems [1,2], Output error systems, Box–Jenkins systems [5] and ARX-like systems [6]. A lot of parameter estimation methods based on least squares (LS) principle, have been present, such as instrumental variable methods [1], multi-innovation identification theory [2], prediction error (PE) methods [7], stochastic gradient algorithm [8].

The aforementioned systems have a common feature, that is the assumed disturbance only occurring on the output. The disturbances induced by the actuator dynamic response acting on the input, or measurement error caused by the uncertainty of the measuring mechanism occurring on the input, are often ignored. However, in the field of high technology and high precision, small interference on the system is likely to produce great influence. Actually, there are a class of errors in variable (EIV) systems abounded in engineering application, such as time series econometric model, blind channel equalization in communications, multivariate calibration in analytical chemistry, etc [9]. Both input and output of those systems has noise disturbance, as depicted in Fig. 1. The variables are described as:

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- $u_0(t)$: actual control variable or system input;
- $y_0(t)$: real value of the system output;
- $\xi(t)$: dynamical disturbance or noise acting on the system input;
- $\zeta(t)$: disturbance or noise adding to the system output;
- $y(t)$: observed system output;
- $u(t)$: observed system input or given value of control variable.

Unbiased parameter estimate for EIV system has been an active research topic for several decades because it is important for system modeling and control in practice [10,11], see the works, e.g. Zheng proposed the bias correction based least squares method [10,11], Hong analyzed the accuracy of bias-eliminating least squares method [12], Aguero presented a single theorem for EIV dynamic system identifiability problem [13], Song studied the identification problem of errors-in-variables systems with nonlinear output observations [14].

In a word, the unbiased parameter estimate approaches above are all based on the strict mathematical assumption that the system input and noise disturbance are stationary and ergodic. It is impossible for system input and noises to completely meet all the conditions in engineering practice application. And few address the recursive identification methods for EIV systems on-line identification. However system input and noises are going to progressively follow asymptotic mathematical assumption behaviors. In recent years, the auxiliary model (AM) [2] based identification approaches get more concerns due to simple concept and low computation [15,16]. It is well-known that transfer function is of significant for both quantitative and qualitative analysis of dynamic systems. The focus of this paper is to study the unbiased parameter estimation for EIV system transfer function based on the auxiliary mode identification idea and under asymptotic mathematical assumptions.

2. Problem formulation

A class of EIV system may have the following transfer function description:

$$G(z) = \frac{z^{-d}\sum_{j=0}^{m}\beta_j z^{-j}}{1+\sum_{i=1}^{n}\alpha_i z^{-i}}, \tag{1}$$

where z^{-1} represents unit back shift $[z^{-1}y(t) = y(t-1)]$, $d \geqslant 0$ is the given time delay, n and m are known system order and input dynamical order. This note mainly deals with the unbiased parameter estimate for transfer function coefficients $[\alpha_i, i = 1, 2, \ldots, n]$ and $[\beta_j, j = 1, 2, \ldots, m]$ from noisy input and output data $\{u(t), y(t), t = 1, 2, \ldots, L\}$, here t represents sampling point on time series sequence and L is the length of time series sequence.

Difference equation for sampled EIV system can be written as

$$[y(t) - \zeta(t)] + \sum_{i=1}^{n}\alpha_i[y(t-i) - \zeta(t-i)] = \sum_{j=0}^{m}\beta_j[u(t-d-j) - \xi(t-d-j)]. \tag{2}$$

Rewrite the aforementioned equation to ARX-like system

$$y(t) = \sum_{i=1}^{n}\alpha_i y(t-i) + \sum_{j=0}^{m}\beta_j u(t-d-j) + v(t), \tag{3}$$

$$v(t) = \sum_{i=1}^{n}\alpha_i\zeta(t-i) - \sum_{j=0}^{m}\beta_j\xi(t-d-j) + \zeta(t). \tag{4}$$

Define the parameter vector $\boldsymbol{\theta}$ and the information vector $\boldsymbol{\varphi}(t)$ as

$$\boldsymbol{\theta} := [\alpha_1,\ \alpha_2,\ \ldots,\ \alpha_n,\ \beta_0,\ \beta_1,\ \ldots,\ \beta_m^{\dagger}]^{\mathrm{T}} \in \mathbb{R}^N,\ N = m + n,$$

$$\boldsymbol{\varphi}(t) := [-y(t-1), \ldots, -y(t-n), u(t-d), \ldots, u(t-d-m)]^{\mathrm{T}} \in \mathbb{R}^N,$$

then the ARX-like model in (3) can be rewritten in a regressive form:

$$y(t) = \boldsymbol{\varphi}^{\mathrm{T}}(t)\boldsymbol{\theta} + v(t). \tag{5}$$

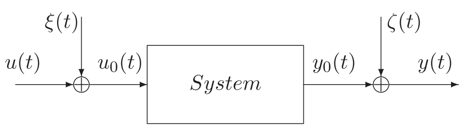

Fig. 1. The single-input single-output EIV system.