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Modeling and numerical sensitivity study on the conjecture of a subglacial lake at Amundsenisen, Svalbard

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ABSTRACT

We present a new numerical procedure to assess the plausibility of a subglacial lake in case of relative small/moderate extension and surging temperate icefield. In addition to the flat signal from Ground Penetrating Radar remote survey of the area, early indication of a likely subglacial lake, required icefield data are: top surface elevation and bathymetry, top surface velocity at some points, in-depth temperature and density profiles of upper layer. The procedure is based on a mathematical model of the evolution of dynamics and thermo-dynamics of the icefield and of a subglacial lake. The Glen's law is adopted for ice rheology and Stokes reduction is applied; Large Eddy Simulation technique is used for the lake. Ice/water phase change is described. Finite volumes for model discretization and a front-tracking technique to follow the moving interface characterize the numerical method.

We have applied this procedure to the case of a subglacial lake conjectured in the area of Amundsenisen, Svalbard, and, here, the results of a sensitivity study are discussed. In particular we point out that the effect of firn and snow upper layers on the system, in terms of temperature field, density and water content, has to be included in the modeling, as it contributes to the overcoming of the ice metastable state and the release of subglacial water. Accordingly, ice water content changes have to be carefully described. The depth of the bed depression is confirmed to be critical for the formation of the lake.

1. Introduction

The fact that icy moons of the solar system, such as Europa and Enceladus, are known to have oceans of water beneath their thick ice crusts, has further increased the importance to study the polar subglacial hydrology, for the similarities between the two environments and the possible extended use of the gained knowledge [1,2]. The attention is, here, focused on polar subglacial lakes.

In Antarctica about 379 subglacial lakes (water basins under an icefield lying on rocky bottom) have, now, been identified [3], mostly via satellite images and airborne radio echo-sounding data where they are revealed like flat regions on the ice-sheet surface. In many cases such flat areas have subglacial lakes beneath them as long as [4] (i) the reflection from the ice-sheet base is stronger than adjacent ice-bedrock reflections, (ii) echoes of constant strength occur along the track, indicating that the surface is very smooth, (iii) echoes from the inside of the conjectured lake detect very flat and horizontal

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(mirror-like) surface with slopes less than 1%. Nevertheless a water film on a flat rocky bottom has very similar characteristics and might be interpreted erroneously as being a sub-glacial lake.

The principal aim of this work is to provide a new mathematical numerical tool for seeking additional hints on the plausibility of the existence of a subglacial lake based on the equations of the dynamics and thermo-dynamics of the grounded icefield and a conjectured subglacial lake.

Incorporating the appropriate ice constitutive law (Glen's law) (see Paterson [5]), we have extended to this application well-tested modeling and numerical simulation tools developed by two of the authors for solving moving boundary problems in materials science (metal melting/solidification, artificial crystal growth) [6–8] and for the evolution of the icy crust of Europa [9].

We have considered the case of the Amundsenisen Plateau at Southern Spitzbergen (Svalbard archipelago), an icefield of about 80 km^2 in area, where radio-echo sounding measurements show high intensity returns from a nearly flat basal reflector at four zones, all of them with ice thickness larger than 500 m [10]. These reflections might correspond to subglacial lakes. In order to determine whether, in one of those zones, a subglacial lake is compatible with other measured quantities, such as density and temperature of ice upper layer and ice surface velocities, we propose to simulate the system at fixed measured conditions, lake included, by solving numerically the model introduced and, then, check the compatibility of the results.

This paper is organized as follows: in Section 2, the mathematical model is introduced; in Section 3, the finite volume solution procedure is sketched; in Section 4, the physical problem at Amundsenisen is described and the simulation plan for the conjecture check is proposed; in Section 5, the numerical results of sensitivity tests are presented and discussed and, in Section 6, conclusions are drawn.

2. Mathematical modeling

A prototypical space geometry, as in Fig. 1, is considered, that is a 2D section with an icefield, flowing from left to right on a rocky bottom, and a subglacial lake, that has been conjectured under the icefield in correspondence of a bed depression. Depending on external temperature history, the interface of ice with atmosphere might have very complex rheology in correspondence of the snow layer and the firn layer, the last one formed from water and ice sublayers resulting from successive refreezing events with compression.

The mathematical model adopted is based on the classical mechanics laws and principles: for the icefield, we refer the reader to Greve and Blatter [11] and, for phenomenological aspects of glacier physics, to Paterson [5]; in addition, references focussed on subglacial lakes and taken into account in this work, are Thoma et al. [12–14].

2.1. Icefield equations

Ice crystals have a structure that allows continuous deformation in response to an applied shear stress (creep, fluid-like behavior) in a way that is dependent on the direction of stress relative to the crystal planes (anisotropy). However ice consists of a huge amount of crystallites (polycrystalline ice) and the single crystal anisotropy averages out in the compound which exhibits an isotropic macroscopic behavior. Elastic deformation, primary, secondary and tertiary creep are the subsequent deformation phases that a polycrystalline ice sample undergoes due to constant shear stress. Such a non-newtonian rheology is satisfactorily described by the Glen's law, a power law of the deformation gradient components for the effective viscosity of ice μ_i [5,15], that is

$$\mu_i = 2^{-(1+n)/2n} A^{-1/n} \left\{ tr(D^2) \right\}^{1-n/2n} \tag{1}$$

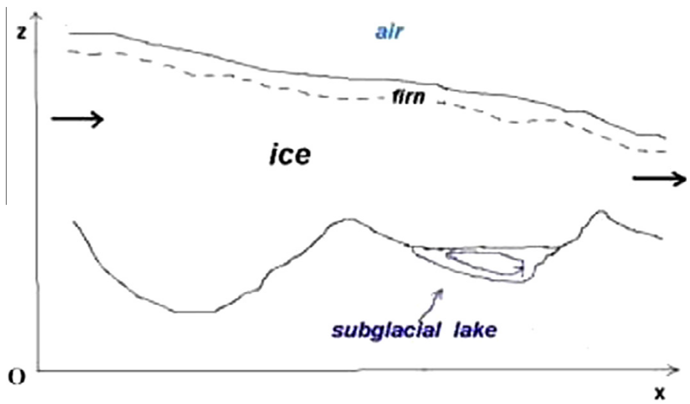

Fig. 1. Prototypical problem geometry.