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Combined electroosmosis-pressure driven flow and mixing in a microchannel with surface heterogeneity

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ABSTRACT

We investigate the combined pressure-driven electroosmotic flow near a wall roughness in the form of a rectangular block mounted on one wall of an infinitely long microchannel. The insulated rectangular block has a constant surface potential which is different from the surface potential of the remaining part of the channel walls. The characteristics for the electrokinetic flow are obtained by numerically solving the Navier–Stokes equations coupled with the Nernst–Planck equation for ion transport and the Poisson equation for electric field. Vortical flow develops above the block when the surface potential of the block is in opposite sign to that of the surface potential of the channel. The strength of the re-circulating vortex grows as the surface potential of the block increases. Vortical flow also depends on the Debye length when it is in the order of the channel height. The combined effects due to the geometrical modulation of the channel wall and heterogeneity in surface potential is found to produce a stronger vortex and hence a stronger mixing, as compared with the effect of either of these. The recirculating vortex, which appears on the upper face of the block, grows and the average electroosmotic velocity increases with the increase of the electrolyte concentration. The recirculating vortex does not appear when EDL is thick and the effect of over-potential of the block on electroosmosis is negligible. The loss of momentum near the obstacle is compensated by the electrostatic force near the EDL, which prevents flow separation upstream or downstream of the obstacle. The mixing performance of the present configuration is compared with several other cases of surface modulation. The impact of the imposed pressure gradient on the vortical flow due to surface heterogeneity is analyzed.

1. Introduction

One prominent difference between fluid motions in a micro/ nano-channel and the flow in a macrochannel is strong fluid-wall interactions in the former case. As the channel size decreases, the surface-to-volume ratio increases. Various properties of the walls, such as surface roughness of the order of nanometer and/ or heterogeneity in surface potential can therefore greatly affect the fluid motions in microfluidics. An electric double layer, consisting of a Stern layer and a diffuse layer, is formed near a charged wall. Under the action of an electric field, tangential to the solid surface, the surplus ions in the diffused double layer experiences a Columbic force and starts moving towards the electrode, which in turn results in an

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electroosmotic flow (EOF) in a channel. Thus, an alteration of the surface property can greatly affect the electric double layer characteristics and thereby the electroosmosis.

In a macroscopic fluidic device, turbulent flow can be generated to enhance a mixing process with an acceptable mixing time period. However, most microfluidic systems are inherently limited to a low Reynolds number which corresponds to a laminar flow. Because of this, mixing of fluid in miniaturized bio-analytical instruments is dominated by the diffusion effect. Techniques based on different actuation methods to enhance micro-mixing have been discussed by several authors. These techniques can be broadly classified in two categories namely, active mixing and passive mixing [1,2]. Passive mixers typically use particular channel geometry configurations to increase the interfacial area between the liquids to be mixed. The active mixers introduce moving parts inside microchannels or apply an AC-electric field.

The electroosmotic flow strongly depends on the surface charge density or ζ-potential. The enhancement of mixing in microfluidics has been demonstrated by several authors [3–7] through surface potential modulation. The motivation behind the surface potential heterogeneity is to induce a transverse flow, which in turn enhances the interfacial area of the species to be mixed. Bhattacharyya and Nayak [6] and Chen and Conlisk [7] demonstrated a technique to enhance mixing in microfluidics by introducing a potential patch on the wall of a micro/nano channel. There they have cited most of the recent studies on improved electrokinetic mixing through surface potential heterogeneity. Jain and Nandakumar [8] studied the optimal heterogeneous surface pattern to enhance micromixing. The EOF in a heterogeneous microchannel under a periodic electric field was considered by Lin and Chen [9]. Foregoing studies suggest that if the patch potential is of opposite sign to that of the channel wall potential, the flow separates from the channel wall and a recirculation vortex develops above the patch. Most of the studies on EOF with surface potential heterogeneity, except [6,7], is based on the free-slip model. In the free-slip model, the electrolyte is considered to be electrically neutral out side the EDL and the ion distribution is governed by the equilibrium Boltzmann distribution. The non-uniformity in wall potential of the channel creates a non-uniform fluid flow and hence induces a pressure gradient everywhere in the flow. The advection effect and non-neutrality of the electrolyte near a potential patch makes the equations for fluid flow and ion transport coupled. Thus, the assumption of Boltzmann distribution for ions, which neglects the effects of fluid convection and electromigration due to imposed field, may not be valid.

The surface roughness of the order of a few angstroms can have a big impact on flows in microfluidic or nanofluidic channels [10]. Generally, microchannel surfaces exhibit certain degrees of roughness generated by the manufacturing techniques or adhesion of biological particles from the liquids. The height of the surface roughness usually ranges from a few nanometers to micrometers. The geometric modulation of a channel wall to increase the interfacial area between the liquids to be mixed was studied by Ramirez and Conlisk [11]. Datta and Ghosal [12] made an asymptotic analysis to study the transport of a solute in a straight microchannel of axially variable cross-sectional shape. Hu et al. [13] have analyzed the effect of surface roughness and grooved microchannels on electroosmotic flow. The effect of undulation of a charged surface on the electroosmotic flow near the surface is analyzed by Lin [14,15]. It is found from those studies that the undulation of surface may increase the magnitude of flow due to the increment of the total fixed charges on undulated surfaces. Bhattacharyya and Nayak [16] found that the surface modulation involving geometric and surface potential heterogeneity causes a stronger convection effect in nanochannels. There they have compared the results based on the non-linear Nernst–Planck model with the results obtained by considering the equilibrium Poisson–Boltzmann model and found a large discrepancy near the surface modulation. The groove-based microchannel to promote mixing was considered by Jain et al. [17].

Microvortices have an advantage in species mixing in microdevices. In some situations, the appearance of vortices need to be suppressed so as to avoid aggregation of suspended particles leading to the eventual jamming of the device. We have investigated the control of vortical flow through an imposed pressure-driven flow. The EOF may be accompanied by a pressure driven flow due to a pressure difference along the longitudinal direction of the channel. The pressure gradient may arise due to a leakage or a difference in the column height between the reservoirs connected by the channel [18]. Separation of non-neutral electrolytes in a charged microchannel by using a pressure gradient in combination of electroosmotic flow is made by Dutta [19]. A combined pressure-driven and electroosmotic flow is used to separate ionic species in a nanofluidic channel by Gillespie and Pennathur [20]. In a recent paper, Song et al. [21] have studied analytically the convection–diffusion of an analyte in a rectangular channel under EOF and pressure driven flow. There they have considered the effect of both negative as well as a positive pressure gradient. The effect of fluid inertia on EOF in a microchannel with nonuniform surface potential is analyzed by Bhattacharyya and Bera [22] through the Nernst–Planck model.

We focus on the combined pressure and electrokinetic effects on the formation of vortical flow in a microchannel in the vicinity of a wall mounted rectangular block of height d, a fraction of the channel height. The ζ-potential along the block is considered to be of opposite sign to that of the ζ-potential of the homogeneous parts of the wall. The geometrical modulation and surface potential heterogeneity makes the equations for fluid flow, electric field and ion distribution coupled. The characteristics for the electrokinetic flow are obtained by numerically solving the Poisson equation, the Nernst–Planck equation, and the Navier–Stokes equations, simultaneously.

2. Mathematical Model

We consider a long rectangular channel of height h and width W with $h \ll W$ filled with an incompressible Newtonian electrolyte of uniform permittivity (ϵ_e) and viscosity (μ). An obstacle in the form of a rectangular block of length (l) and height (d) is considered to be mounted along the lower wall of the channel (Fig. 1). The external applied electric field is