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Parametric instability of flexible rotor-bearing system under time-periodic base angular motions

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ABSTRACT

Parametric instability of flexible rotor-bearing system under time-periodic base angular motions is analyzed in this paper. The accurate finite element model for the flexible rotor-bearing system under time-varying base angular motions is derived based upon the energy theorem and Lagrange's principle. Three base angular motions, including the rolling, pitching and yawing motions, are assumed to be sinusoidal perturbations superimposed upon constant terms. Considering the time-varying base movements, the second order differential equations of the system will have time-periodic gyroscopic and stiffness coefficients. The discrete state transition matrix (DSTM) method is introduced for numerically acquiring the instability regions. Based upon these, instability computations for a rotor-bearing system with one base motion alone and two base motions together are conducted, respectively. The effects of rotating speed, amplitudes of base motion and phases between two base motions on both the primary and combination instability regions are discussed in detail.

1. Introduction

Rotor dynamic problems have been sufficiently studied during the past decades and much attention has been paid to the rotor systems under fixed supports [1–4]. Turbo-machinery such as turbines, pumps and compressors, which are installed in transportation systems, are examples of rotors on moving bases. Such rotor system might undergo large time-varying linear as well as angular displacements. The moving base not only causes external force excitations but also gyroscopic and stiffness excitations to the rotor-bearing system. Over the past ten years, a number of studies [5–22] have been carried out on the dynamic characteristics of base excited rotor systems, which differ distinctly from that of the fixed supported rotor systems.

Lin and Meng [5] first studied the dynamics of a Jeffcott rotor carried within a maneuvering aircraft. The results showed that the rotor's unbalanced response is obviously influenced by the flying status of the aircraft. Zhu and Chen [6] also carried out extensive studies on the vibrational characteristics of flexible rotors during maneuvering flight in aero-engine system. Lee et al. [7] presented a generalized finite element modeling method of a rotor-bearing system considering a base-transferred shock force. The state-space Newmark method of a direct time integration scheme based on the average velocity concept was utilized to obtain the transient response. The whirling orbits were described analytically and experimentally by Driot et al. [8]. Simple rotor model (two degrees of freedom) was developed and the multiple scales method was utilized to

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obtain analytical expressions. Later, the studies were extended to the stochastic dynamic analysis of an asymmetrical rotor subjected to random base translational motions [9]. El-Saeidy and Sticher [10] obtained the responses of a rigid rotor-bearing system subjected to rotating mass unbalance plus harmonic base excitations. Saha et al. [11] conducted analysis of a uniformly spinning shaft with a non-central disc mounted on a rotating (precessing) base, where the spin axis and the precession axis intersect at right angle. Very recently, Dakel et al. [12] presented an improved finite element model, in which both the time-periodically base motions and geometric asymmetric shaft and disk were taken into account. The influence of rotational or translational motions of the support was analyzed by means of orbits of the rotor, responses in the time-domain and fast Fourier transforms. The above studies concerned about the rotor system without faults. Lin et al. [13], Jing et al. [14] and Yang et al. [15], respectively, studied the effect of transverse cracks on the nonlinear dynamics of a rotor system with moving bases. Recently, the authors [21] also analyzed the dynamic response of cracked rotor-bearing system under time-dependent base movements. Through sensitivity analysis, the unique spectra for detecting open and breathing transverse cracks in base excited rotating machinery were given. Based upon these, the studies were extended to geared rotor system under sinusoidal moving bases [22]. Hou et al. [16] focused on the nonlinear vibration phenomenon caused by aircraft hovering flight in a rub-impact rotor system supported by two general supports with cubic stiffness. Moreover, several researchers also concerned about the active vibration control of base excited rotor-bearing system utilizing the electromagnetic actuators [17–19].

As the literature shows, extensive efforts have been devoted to analyze the dynamic responses and vibration control problems, while the parametric instability induced by time-dependent base angular motions has not gained sufficient attentions. Actually, the parametric stiffness and gyroscopic excitations from the moving base causes instability and severe vibration under certain operating conditions. Determination of operating conditions of parametric instability is crucial to the design and usage of the base excited rotor system. Duchemin et al. [20] observed the parametric instability of a flexible rotor system under a sinusoidal support rotation. As the simple rotor model was used, so their study could only give a qualitative insight into the systems's unstable behavior. Das et al. [18,19] utilized the electromagnetic actuators to control the unstable responses of unbalanced flexible-shaft systems parametrically excited due to base motion. Only the pitching base motion was considered, while the other two base motions (rolling and yawing) were ignored in their studies. In fact, the parametric instability regions induced by the three base angular motions alone might be different from each other. Besides, when two time-dependent base motions are in operation simultaneously, there would be two sources of parametric excitations with various amplitudes and phases. In this case, the instability behaviors of the base excited rotor system might have some distinct features. Research in this area has not been reported before.

Thus, the parametric instability of flexible rotor-bearing system under time-periodic base angular motions is analyzed in this paper. The accurate finite element model for the flexible rotor-bearing system under time-varying base angular motions is derived based upon the energy theorem and Lagrange's principle. Three base angular motions, including the rolling, pitching and yawing motions, are assumed to be sinusoidal perturbations superimposed upon constant terms. Considering the time-varying base movements, the second order differential equations of the system will have time-periodic gyroscopic and stiffness coefficients. The discrete state transition matrix (DSTM) method [23] is introduced for numerically acquiring the instability regions. Based upon these, instability computations for a rotor-bearing system with one base motion alone and two base motions together are conducted, respectively. The effects of rotating speed, amplitudes of base motion and phases between two base motions on both the primary and combination instability regions [24] are discussed in detail. Finally, some conclusions are presented.

2. Finite element modeling of flexible rotor-bearing system under time-dependent base angular motions

The rotor base is assumed to be sufficiently rigid. Three co-ordinate systems are defined in the paper: inertial frame of reference ($X_0 - Y_0 - Z_0$), frame of moving base ($X_b - Y_b - Z_b$) and rotor frame ($X - Y - Z$, fixed to the shaft). The rotor shaft rotates about its own axis with constant speed Ω_s. Without loss of generality, the mass center (also the origin of $X_b - Y_b - Z_b$) of the base is also assumed to be at the left bearing, as shown in Fig. 1. Three angular rotations of the base about frame

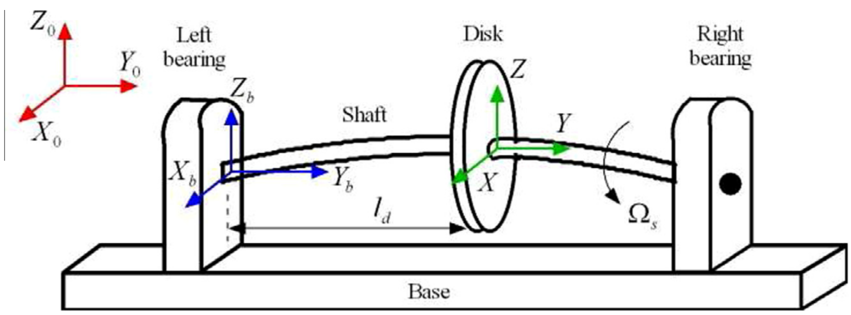

Fig. 1. Rotor-bearing system on moving base with various co-ordinate systems.