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Short communication

Quasi-static axially symmetric viscoplastic flows near very rough walls

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ABSTRACT

The paper deals with asymptotic behavior of viscous and viscoplastic solutions in the vicinity of very rough walls under conditions of axial symmetry. The constitutive equations adopted include a saturation stress. A distinguished feature of this model is that the regime of sticking at the wall may be incompatible with other boundary conditions. In this case the regime of sliding must occur and solutions are singular in the vicinity of such surfaces. The exact asymptotic representation of the singular solutions is controlled by the dependence of the equivalent stress on the equivalent strain rate as the latter approaches infinity. There exist such dependences that the viscoplastic model possesses a smooth transition of qualitative behavior between rigid perfectly plastic and viscoplastic solutions, and this may prove to be advantageous for some applications.

1. Introduction

Rigid perfectly plastic solutions are singular in the vicinity of maximum friction surfaces at sliding [1]. In particular, the quadratic invariant of the strain rate tensor approaches infinity near such surfaces. In the case of rigid perfectly plastic solids, the maximum friction surface is defined by the condition that the friction stress at sliding is equal to the shear yield stress. The main result obtained in [1] has been extended to the double shearing model and a model of anisotropic plasticity in [2,3], respectively. A description of the double shearing model is given in [4] and the model of anisotropic plasticity in [5]. For various models, there are numerous analytic and semi-analytic solutions that are singular (although this feature of solution behavior is not always mentioned by the authors) [[6–14] among many others]. The models adopted in these works are rate – independent. It has been shown in [15] that no singular solutions of the aforementioned type are possible in the case of conventional rigid viscoplastic models. Available semi-analytical solutions for such models found with the use of a more general friction law than the maximum friction law do not exist when this general law reduces to the maximum friction law [16–19]. Meanwhile, the singular solutions near maximum friction surfaces can play a significant role in the development of methods for predicting the evolution of material properties in a narrow layer near surfaces with high friction in metal forming processes [20]. Such layers often appear in special experiments and real processes [21–25]. Velocity gradients are very high in these layers. Conventional numerical methods fail to predict both high velocity gradients and the evolution of material properties in the vicinity of interfaces between tool and workpiece with high friction [21,26,27]. A possible reason for that is that solutions may be singular [1–3]. It is therefore important to find the exact asymptotic representation of solution behavior near maximum friction surfaces. Such representations can be then used in the extended finite element

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method [28] to overcome the aforementioned difficulty with the use of standard finite element techniques. In particular, it has been shown in [29,30] that solutions for viscoplastic models with a saturation stress may be singular near maximum friction surfaces. The meaning of the saturation stress is that the tensile stress in a hypothetical tensile test approaches its value as the corresponding strain rate approaches infinity. In the present paper, the exact asymptotic representation of the solution for such models is derived in the vicinity of maximum friction surfaces under conditions of axial symmetry assuming that the solution can be represented by power series.

2. System of equations

The elastic component of strain is disregarded. An instantaneous state of stress and velocity is studied in the vicinity of very rough walls. In an axially symmetric distribution of stress the non-vanishing components are σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} and σ_{rz}, referred to cylindrical coordinates (r, θ, z) with z as the axis of symmetry. The non-vanishing velocity components are u_r and u_z, respectively perpendicular and parallel to the axis. The stress and velocity are independent of θ, and are functions only r and z. Therefore,

$$\xi_{\theta\theta} = \frac{u_r}{r}. \tag{1}$$

Here $\xi_{\theta\theta}$ is the circumferential strain rate. Consider an arbitrary smooth curve L in a generic meridian plane (Fig. 1). This curve represents the surface of a wall near which material flow occurs. With no loss of generality the wall is regarded as motionless. It is convenient to introduce a local Cartesian coordinate system (x, y) in the meridian plane whose origin is taken to be situated at an arbitrary point, M, of curve L. The y-axis is directed along the normal to L, away from the wall and towards the viscoplastic material. The x-axis is directed opposite to the velocity vector $\mathbf{u}$ at M. Let σ_{xx}, σ_{yy} and σ_{xy} be the components of the stress tensor referred to the Cartesian coordinates. Then, the equilibrium equations in these coordinates are

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + Q_1 = 0, \quad \frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + Q_2 = 0, \tag{2}$$

where Q_1 and Q_2 are independent of stress derivatives. The constitutive equations consist of the yield criterion and the flow rule. The yield criterion is assumed to be

$$\sigma_{eq} = \Phi(\xi_{eq}), \tag{3}$$

where σ_{eq} is the equivalent stress and ξ_{eq} is the equivalent strain rate. The flow rule is

$$\xi_{xx} = \lambda\tau_{xx}, \quad \xi_{yy} = \lambda\tau_{yy}, \quad \xi_{\theta\theta} = \lambda\tau_{\theta\theta}, \quad \xi_{xy} = \lambda\tau_{xy}, \tag{4}$$

where ξ_{xx}, ξ_{yy} and ξ_{xy} are the components of the strain rate tensor; τ_{xx}, τ_{yy} and $\tau_{\theta\theta}$ are the deviatoric stress components; and λ is a non-negative multiplier. The equivalent stress and the equivalent strain rate are defined as

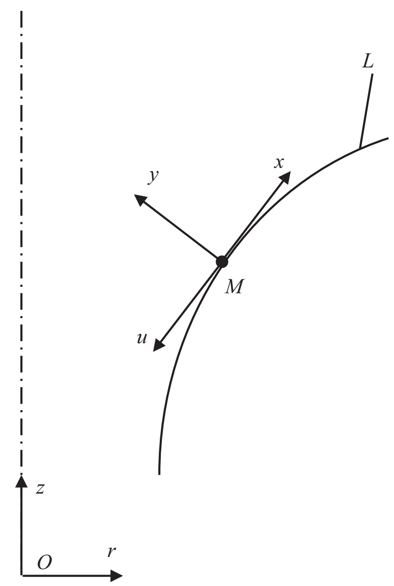

Fig. 1. Local Cartesian coordinate system at generic point M on maximum friction surface.