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Analysis of a delayed stochastic predator–prey model in a polluted environment ☆

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ABSTRACT

In this paper, we investigate two-species Lotka–Volterra delayed stochastic predator–prey systems, with and without pollution, denoted by (M) and (M_0), respectively. We show that there exists a unique non-negative solution in each system that is permanent in time average under certain conditions. Moreover, the non-permanence of model (M) is studied. Finally, computer simulations are carried out to verify our results.

1. Introduction

This paper is written as one of our research projects on investigating the dynamics of the Lotka–Volterra (briefly, L–V) stochastic predator–prey system [1–3]. The world economy has grown faster and faster, but the quality of our living environment has declined. With a growing number of toxicant and contaminants entering into the ecosystem, we must address the serious problem of pollution. Toxicant in the environment threaten the survival of exposed species, including ourselves. Therefore, the study of the permanence of species in polluted environments has become more important.

Several studies have examined the effects of toxicant from industrial and household sources on exposed species [4–11]. These studies using deterministic models have provided us with useful bases for protecting species. However, population systems are often subject to environmental noise which has a significant effect on these systems. And, there are many authors who have studied the dynamics of stochastic models [1–3,12–16]. In particular, Liu and Wang investigated the dynamics of stochastic single-species models in polluted environments and obtained acute thresholds between a population's local extinction and (stochastic) weak persistence in the mean [17,18]. However, there has not been much research on the delayed L–V model in polluted environments.

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In this paper, we consider a stochastic delayed predator–prey model with toxicant input. In Section 2, we prepare basic notions and introduce a new stochastic model (M) of predator–prey systems. We show that there exists a unique positive solution to the system in Section 3. Unlike the deterministic system, the stochastic system does not have an interior equilibrium. Therefore, we cannot discuss the stability of the deterministic system. In Section 4, we show that the solution to the system without pollution will tend to a point in time average, and the system with pollution will be stochastic permanent in time average. We give some sufficient conditions for non-permanence of the system (M) in Section 5. In Section 6, we present numerical simulations to illustrate our mathematical findings.

2. The basic models and notations

In this section, we prepare some basic notions on Lotka–Volterra (L–V) stochastic predator–prey system. We base our model on the following [19]:

$$\begin{cases} dx_1(t) = x_1(t)[r_{10} - r_{11}C_1(t) - a_{11}x_1(t) - a_{12}x_2(t)]dt, \\ dx_2(t) = x_2(t)[-r_{20} - r_{21}C_2(t) + a_{21}x_1(t) - a_{22}x_2(t)]dt, \\ dC_1(t) = [k_1C_E(t) - (g_1 + m_1)C_1(t)]dt, \\ dC_2(t) = [k_2C_E(t) - (g_2 + m_2)C_2(t)]dt, \\ dC_E(t) = [-hC_E(t) + \widetilde{C}(t) + u(t)]dt, \end{cases} \tag{2.1}$$

where $\widetilde{C}(t) = -k_1C_E(t)x_1(t) - k_2C_E(t)x_2(t) + g_1C_1(t)x_1(t) + g_2C_2(t)x_2(t)$. These differential equations denote the population growth of a predator–prey system for two species in a polluted environment. Here, $x_1(t)$ and $x_2(t)$ express the densities of the two species at time t respectively, where the parameters r_{i0}, a_{ij} $(i,j = 1,2)$ are all positive, $C_1(t), C_2(t)$ and $C_E(t)$ represent the concentrations of the toxicant in the organism of the prey species, the predator species and the environment, respectively, at time t, r_{10} expresses the intrinsic growth rate of the prey and r_{20} represents the death rate of the predator, r_{11} and r_{21} represent the dose–response of the species prey and predator to the organismal toxicant, respectively. $k_i,\ -g_i,\ -m_i$ $(i = 1,2)$ express the absorbing rates of the toxicant from the environment, excretion and depuration rates of the toxicant, respectively. $-hC_E(t)$ represents the loss rate of the toxicant due to volatilization, $-k_1C_E(t)x_1(t)$ and $-k_2C_E(t)x_2(t)$ represent the toxicant uptake by the organisms, $g_1C_1(t)x_1(t) + g_2C_2(t)x_2(t)$ denotes the rate of increase of the toxicant due to the excretion of both populations, and $u(t)$ is the exogenous rate of toxicant input into the environment, which is assumed to be bounded and $0 \leqslant u(t) \leqslant u_1 < +\infty$. To simplify model (2.1), we will suppose that the capacity of the environment is so large that the change of toxicants in the environment, which comes from uptake and digestion by the organisms, can be neglected [7], that is, $\widetilde{C}(t) = 0$. It follows that Eq. (2.1) can be rewritten as:

$$\begin{cases} dx_1(t) = x_1(t)[r_{10} - r_{11}C_1(t) - a_{11}x_1(t) - a_{12}x_2(t)]dt, \\ dx_2(t) = x_2(t)[-r_{20} - r_{21}C_2(t) + a_{21}x_1(t) - a_{22}x_2(t)]dt, \\ dC_1(t) = [k_1C_E(t) - (g_1 + m_1)C_1(t)]dt, \\ dC_2(t) = [k_2C_E(t) - (g_2 + m_2)C_2(t)]dt, \\ dC_E(t) = [-hC_E(t) + u(t)]dt. \end{cases} \tag{2.2}$$

The above model does not incorporate the effect of time delay, which is always present [20–22]. For a long time, it has been recognized that delays can have a complicated impact on the dynamics of a system. For example, delays can cause the loss of stability and can induce various oscillations and periodic solutions. Incorporating the effect of time delay, Eq. (2.2) with two same time delay in the interaction terms can be rewritten as:

$$\begin{cases} dx_1(t) = x_1(t)[r_{10} - r_{11}C_1(t) - a_{11}x_1(t) - a_{12}x_2(t-\tau)]dt, \\ dx_2(t) = x_2(t)[-r_{20} - r_{21}C_2(t) + a_{21}x_1(t-\tau) - a_{22}x_2(t)]dt, \\ dC_1(t) = [k_1C_E(t) - (g_1 + m_1)C_1(t)]dt, \\ dC_2(t) = [k_2C_E(t) - (g_2 + m_2)C_2(t)]dt, \\ dC_E(t) = [-hC_E(t) + u(t)]dt. \end{cases} \tag{2.3}$$

In the real world, however, the natural growth of many populations is inevitably affected by some random disturbance. In this paper, we assume that parameters r_{10} and r_{20} of Eq. (2.3) are perturbed with

$$r_{10} \to r_{10} + \alpha_1\dot{B}_1(t), r_{20} \to r_{20} + \alpha_2\dot{B}_2(t),$$

where $B_i(t)$ $(i = 1,2)$ are mutually independent one-dimensional standard Brownian motions with $B_i(0) = 0$ and $\alpha_i > 0$ $(i = 1,2)$ being the intensities of white noises. It follows that the stochastic version corresponding to the deterministic system with time delay takes the following form:

$$(M)\begin{cases} dx_1(t) = x_1(t)[r_{10} - r_{11}C_1(t) - a_{11}x_1(t) - a_{12}x_2(t-\tau)]dt + \alpha_1x_1dB_1(t), \\ dx_2(t) = x_2(t)[-r_{20} - r_{21}C_2(t) + a_{21}x_1(t-\tau) - a_{22}x_2(t)]dt - \alpha_2x_2dB_2(t), \\ dC_1(t) = [k_1C_E(t) - (g_1 + m_1)C_1(t)]dt, \\ dC_2(t) = [k_2C_E(t) - (g_2 + m_2)C_2(t)]dt, \\ dC_E(t) = [-hC_E(t) + u(t)]dt. \end{cases}$$