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Interval optimization based line sampling method for fuzzy and random reliability analysis

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ABSTRACT

For structural system with fuzzy variables as well as random variables, a novel algorithm for obtaining membership function of fuzzy reliability is presented on interval optimization based Line Sampling (LS) method. In the presented algorithm, the value domain of the fuzzy variables under the given membership level is firstly obtained according to their membership functions. Then, in the value domain of the fuzzy variables, bounds of reliability of the structure are obtained by the nesting analysis of the interval optimization, which is performed by modern heuristic methods, and reliability analysis, which is achieved by the LS method in the reduced space of the random variables. In this way the uncertainties of the input variables are propagated to the safety measurement of the structure, and the membership function of the fuzzy reliability is obtained. The presented algorithm not only inherits the advantage of the direct Monte Carlo method in propagating and distinguishing the fuzzy and random uncertainties, but also can improve the computational efficiency tremendously in case of acceptable precision. Several examples are used to illustrate the advantages of the presented algorithm.

1. Introduction

The uncertainties affecting the performance of structural systems need to be quantified and propagated to obtain structural reliability. For many years, these uncertainties have been considered as randomness, and statistics and probability theory have been regarded the only way to deal with them and to perform reliability analysis of structures. Typically, statistics is used to obtain, from the available set of data, parameters which define the occurrence properties of uncertain variables which are random in nature; while probability converts these information to occurrence functions (probability density functions–PDFs–and cumulative density functions–CDFs) and defines the general framework for reliability analysis [1]. Based on mathematical statistics and probability theory, many uncertainty propagation methods such as numerical simulation method, first-order reliability method, importance sampling method (IS) etc., have been established and received much attention in the reliability literature.

However, probability can be consistently used only when input variables are random in nature and precise information on their variation are available in order to define their PDFs. This is not usually the case in structural design, since statistical information on certain variables, e.g., loads and resistances, may be very scarce or even absent. In this situation, sufficient data is not available for defining a probability distribution. Thus, the aforementioned reliability methods for the randomness are often not justified for the design of real structures [1].

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Due to these reasons, many non-probabilistic methods, such as interval analysis, possibility theory and fuzzy set theory, have been developed in recent years. The main goal is to derive computationally less expensive algorithms not requiring a priori assumption of PDFs of input variables and giving conservative values for safety factors [2]. Among these methods, fuzzy set theory is known as a versatile and effective tool in non-probabilistic uncertainty analysis. In situations where sufficient information is not available for defining a probability distribution, fuzzy set theory can be used to represent the available data in an analytical form, and model this type of uncertainty as fuzzy variables. Based on the fuzzy set theory, some reliability methods have been presented to deal with the fuzzy variables. Briabant et al. [3] presented a possibilistic approach for structural design optimization for non-random variables. In this method, they employed membership functions to describe the fuzzy variables, where the membership levels represent the confidence of the fuzzy variables taking certain interval values. According to this idea, a series of reliability methods have been developed for dealing with fuzzy variables, such as vertex method in literature [4], optimization techniques in literatures [5,6] etc.

All the methods discussed above consider either the random variables or the fuzzy ones, but do not accommodate a combination of these two kinds of variables. Nevertheless, in most engineering structure, information might be available to represent some variables with probability distributions and some with membership functions. This is typically the case in aircraft design industry where sufficient data is available to model material strength of metallic structures as probability distributions but geometric tolerances are rarely quantified using distributions. The reason lies in that tolerances are slight variations about the nominal values and can be modeled as fuzzy variables to determine the bounds on reliability of the structure. Therefore, only the methods which consider these two kinds of uncertain variables simultaneously can accurately estimate the safe level of the structural system [7–9].

The direct method for propagating the fuzzy and random uncertainties is the Monte Carlo method (MC) [7]. By separating the fuzzy and random variables using a double-sampling framework, the mixed uncertainties can be propagated to the output completely in this method. But the computational cost associated with MC method is usually too expensive to be accepted for complicated structure. Thus, people begin to study highly efficient methods for the propagation of the mixed uncertainties, such as the transformation methods [8,9], iteration method [10] and the method employing efficient sampling technique for random uncertainty in the double-sampling framework [11] etc. However, there are some problems existing in these methods: (1) the transformation methods are limited to the case where no cross items of the fuzzy and random variables are present; (2) The iteration method is based on the first order reliability method. Therefore, it will lose efficiency or become inaccurate in the general case where the structural system involves nonlinear performance functions and/or multiple outputs; (3) the method employing efficient sampling technique for random uncertainty will improve the efficiency of the double-sampling to some extent, but it still depends on screening the membership interval for the fuzzy variables. Therefore, its efficiency is still too low for complicated structure with "black-box" input–output relationship.

By combining the traditional local/global optimization methods with stochastic expansions, Eldred etc., proposed an algorithm for calculating the first two order moments and the reliability index of the output response in the presence of the fuzzy and random uncertainties simultaneously [12]. Due to the combination of interval optimization for fuzzy variables and stochastic expansions for random variables, this algorithm can improve the propagation efficiency of the mixed uncertainties tremendously. However, the optimization methods employed by this algorithm rely on the analytic expressions of the moments of the output response, which are derived from the expansion of the response in the inner loop. For the response metrics whose analytic expressions cannot be derived, such as the structural reliability concerned in the reliability engineering, this algorithm becomes powerless. In addition, calculation of the structural reliability directly from the reliability index is prone to errors. To provide more direct and accurate information for the reliability design and optimization, it is recommended in literature [12] that the IS method [13] can be used to further improve the accuracy of the reliability analysis after obtaining the reliability index. As an improved sampling method, although IS method can improve the efficiency of reliability analysis to some extent, its computational cost is still hard to be accepted in engineering when considering the fuzzy and random uncertainties simultaneously.

Modern heuristic algorithms, such as Simulated Annealing (SA) [14,15] and Genetic Algorithms (GA) [16], are extensively used since their development in 1980s. The reasons lie in that: (1) they do not need the gradient of the function; (2) they are nearly independent of the form of the objective function; (3) they can obtain the global optima with good robustness. As a highly efficient reliability method for high dimensional and small failure probability problems, the advantages of the Line Sampling (LS) method have been fully proved in literatures [17,18] for the case that only normal random variables are involved. In this paper, combining the superiorities of both the modern heuristic algorithms and the LS method, a new algorithm for uncertainties propagation in fuzzy and random reliability analysis is presented. The new algorithm is not only more efficient compared with the existing methods, but also suitable for the case with cross items of the fuzzy variables and the random ones. Furthermore, the new algorithm has more extensive applicability, since it employs the heuristic optimization methods, which usually have few requirements on the form of the objective function.

The remainder of the paper is organized as follows. Section 2 briefly describes the uncertainties propagation from the input variables to the reliability of the structure. The MC-based solution of the propagation is also introduced in this section. The interval optimization based LS method for the uncertainties propagation in the fuzzy and random reliability analysis is detailed in Section 3. Several examples are used in Section 4 to demonstrate the efficiency and accuracy of the presented algorithm. The discussions and the conclusions are given at the end of the paper.