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Distance measures with heavy aggregation operators

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ABSTRACT

The use of distance measures and heavy aggregations in the ordered weighted averaging (OWA) operator is studied. We present the heavy ordered weighted averaging distance (HOWAD) operator. It is a new aggregation operator that provides a parameterized family of aggregation operators between the minimum distance and the total distance operator. Thus, it permits to analyze an aggregation from its usual average (normalized distance) to the sum of all distances available in the aggregation process. We analyze some of its main properties and particular cases such as the normalized Hamming distance, the weighted Hamming distance and the OWA distance (OWAD) operator. This approach is generalized by using quasi-arithmetic means obtaining the quasi-arithmetic HOWAD (Quasi-HOWAD) operator and with norms obtaining the heavy OWA norm (HOWAN). Further extensions to this approach are presented by using moving averages forming the moving HOWAD (HOWMAD) and the moving Quasi-HOWAN (Quasi-HOWMAN) operator. The applicability of the new approach is studied in a decision making model regarding the selection of national policies. We focus on the selection of monetary policies. The key advantage of this approach is that we can consider several sources of information that are independent between them.

1. Introduction

Distance measures are very useful techniques that have been used in a wide range of applications [1–3]. The most common types of distance measures are the total distance and the normalized distance. The first one only considers the sum of all the individual distances considered and the normalized distance considers an average of the individual distances. A very popular distance is the Hamming distance (also known as the Manhattan distance depending on the problems considered). The Hamming distance [3] can be normalized in different ways depending on the interests of the specific problem considered. For example, it is possible to consider the normalized Hamming distance (NHD) and the weighted Hamming distance (WHD) that use the average and the weighted average (WA), respectively.

Recently, several authors have suggested the use of the ordered weighted averaging (OWA) operator in the Hamming distance obtaining the ordered weighted averaging distance (OWAD) operator [4–6]. By using the OWA operator [7–9], we are able to provide a wide range of aggregation operators between the maximum and the minimum. Since its introduction, it has been receiving increasing attention. For example, Merigó and Casanovas extended this approach by using linguistic variables [10]. They also developed a generalization by using induced aggregation operators [11]. Furthermore, they also extended this approach by using the Euclidean distance [12] and the Minkowski distance [13]. Zeng and Su [14]

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studied the use of intuitionistic fuzzy sets in the OWAD operator They also considered imprecise environments with interval numbers, fuzzy information and linguistic generalized aggregation operators [15–17]. Merigó and Gil-Lafuente developed an application in human resource management [18] and in sport management [19]. Yager [20] generalized it by using norms. Merigó et al. [21,22] presented an extension by using similarity measures where the Hamming distance was included as a particular case. Xu and Xia [23] analyzed the use of hesitant fuzzy sets in the OWAD operator and Xu [24] considered fuzzy numbers. Merigó and Yager [25] studied distance measures with moving averages.

An interesting extension is the heavy OWA (HOWA) operator [26,27]. The main advantage is that it allows the aggregation to move between the total operator and the usual OWA operator. Recently, Merigó and Casanovas developed several extensions by using induced aggregation operators [28,29].

In this paper, we present the heavy OWA distance (HOWAD) operator. It is a distance aggregation operator that uses the OWA operator allowing the aggregation to be between the total (or absolute) distance and the normalized (or relative) distance. That is, it uses distance measures in the HOWA operator. Therefore, we get a more complete distance aggregation operator that includes the normalized and the total distance operator. Note that in this paper we use the Hamming distance as the distance measure. But it is also possible to use other distance measures such as the Euclidean distance or the Minkowski distance. We study some of its main properties and particular cases.

We further generalize this approach by using generalized and quasi-arithmetic means obtaining the generalized HOWAD (GHOWAD) operator and the quasi-arithmetic HOWAD (Quasi-HOWAD) operator. Their main advantage is that they include a wide range of particular cases including the HOWAD operator and quadratic HOWAD (HOWQAD) operator.

Moreover, we extend this approach by using norms [20] obtaining the HOWA norm (HOWAN) that include a wide range of particular cases including the addition OWA (A-OWA) and the multiplication OWA (M-OWA). We also extend this approach by using moving averages [25,30] obtaining the heavy ordered weighted moving averaging distance (HOWMAD). Thus, we can analyze the aggregation process in a dynamic way. Moreover, we generalize this model by using generalized aggregation operators and norms forming the quasi-arithmetic heavy ordered weighted moving averaging norm (Quasi-HOWMAN) operator.

We study the applicability of this new approach and we see that it is very broad because we can apply it in a wide range of fields including statistics, economics and engineering. We focus on a decision making problem regarding the selection of strategies. We see that the use of HOWAD operators permits to obtain a more complete representation of the aggregation process when the available information is independent.

This paper is organized as follows. In Section 2 we briefly review some basic concepts. In Section 3 we present the HOWAD operator and study some of its main families. Section 4 presents several generalizations by using generalized aggregation operators, Section 5 by using norms and Section 6 with moving averages. Section 7 develops an illustrative example of the new approach. Finally, in Section 8, we summarize the main conclusions of the paper.

2. Preliminaries

2.1. The Hamming distance

The Hamming distance [2] is a useful technique for calculating the distance between two elements, two sets, etc. In order to define the Hamming distance, first, we will define a distance measure. Basically, a distance measure has to accomplish the following properties.

- Non-negativity: $D(A_1,A_2) \geqslant 0$.
- Commutativity: $D(A_1,A_2) = D(A_2,A_1)$.
- Reflexivity: $D(A_1,A_1) = 0$.
- Triangle inequality: $D(A_1,A_2) + D(A_2,A_3) \geqslant D(A_1,A_3)$.

For example, the weighted Hamming distance (WHD) between two sets $X = (x_1,x_2,\ldots,x_n)$ and $Y = (y_1, y_2,\ldots,y_n)$, can be defined as follows.

Definition 1. A weighted Hamming distance of dimension n is a mapping WHD: $R^n \times R^n \to R$ that has an associated weighting vector W with $w_i \in [0, 1]$ and the sum of the weights is 1, such that:

$$d_{\mathrm{WHD}}(X,Y) = \sum_{i=1}^{n} w_i|x_i - y_i|, \quad (1)$$

where x_i and y_i are the ith arguments of the sets X and Y.

2.2. The OWA operator

The OWA operator was introduced by Yager [7]. It provides a parameterized family of aggregation operators between the maximum and the minimum. It can be defined as follows: