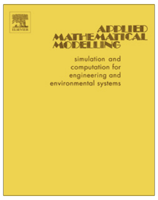
Applied Mathematical Modelling 38 (2014) 3269–3280

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm



Numerical investigation into nonlinear dynamic behavior of electrically-actuated clamped–clamped micro-beam with squeeze-film damping effect

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ARTICLE INFO

Article history:
Received 22 September 2012
Received in revised form 9 August 2013
Accepted 22 November 2013
Available online 22 December 2013

Keywords:
Squeeze film damping effect
Reynolds equation
Pull-in voltage
MEMS
Differential transformation
Finite difference method

ABSTRACT

A numerical investigation is performed into the nonlinear dynamic behavior of a clamped–clamped micro-beam actuated by a combined DC/AC voltage and subject to a squeeze-film damping effect. An analytical model based on a nonlinear deflection equation and a linearized Reynolds equation is proposed to describe the deflection of the micro-beam under the effects of the electrostatic actuating force. The deflection of the micro-beam is investigated under various actuating conditions by solving the analytical model using a hybrid numerical scheme comprising the differential transformation method and the finite difference approximation method. It is shown that the numerical results for the dynamic pull-in voltage of the clamped–clamped micro-beam deviate by no more than 2.04% from those presented in the literature based on the conventional finite difference scheme. The effects of the AC voltage amplitude, excitation frequency, residual stress, and ambient pressure on the center-point displacement of the micro-beam are systematically explored. Moreover, the actuation conditions which ensure the stability of the micro-beam are identified by means of phase portraits. Overall, the results presented in this study confirm that the hybrid numerical method provides an accurate means of analyzing the complex nonlinear behavior of common electrostatically-actuated microstructures.

1. Introduction

Electrostatically-actuated micro-electro-mechanical systems (MEMS) devices are used for many pressure and mass sensing applications [1,2]. Such devices comprise a deformable electrode and a fixed electrode separated by a small air gap. The application of a bias voltage across the two electrodes generates an electrostatic force, which causes the deformable electrode to deflect toward the fixed electrode. At a certain critical value of the bias voltage, the attractive force acting on the deformable electrode exceeds the elastic restoring force, and thus the electrode collapses and makes transient contact with the fixed electrode. This phenomenon is conventionally referred to as the "pull-in phenomenon"; with the corresponding bias voltage designated as the "pull-in voltage". In some application, e.g., pressure sensors [3], the bias voltage must be carefully controlled so as to avoid the pull-in effect. However, in other applications, e.g., optical/radio-frequency (RF) switches [4,5], the bias voltage is deliberately tuned in such a way as to produce a periodic collapse and restoration of the deformable electrode. For the case where the actuating voltage increases only slowly and the inertia force has no effect

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http://dx.doi.org/10.1016/j.apm.2013.11.048

on the mechanical behavior of the electrode, the critical voltage at which the electrode collapses is referred to as the static pull-in voltage (V_{pi}). However, in most practical applications, the actuating voltage varies rapidly, and thus the effects of inertia and damping-related forces must be taken into consideration. Under such conditions, the critical voltage is referred to as the dynamic pull-in voltage (V_{pid}) [6]. In practice, irrespective of the rate at which the actuating voltage varies, it is essential that the voltage at which the pull-in event takes place can be reliably determined in order to ensure that the micro device remains stable over the designated operational range. Chao et al. [7] proposed a novel computational method for predicting the static pull-in event between two micro-plates actuated by a distributed electrostatic force. Cheng et al. [8] used a simple lumped model to predict the static pull-in voltage for rigid and deformable electrostatic micro-actuators. Vogl and Nayfeh [9] presented an analytical reduced-order macromodel for investigating the dynamic behavior of electrically-actuated clamped micro-plates. Hung and Senturia [10] used a macromodel to simulate the dynamic behavior of a clamped–clamped micro-beam within a pressure sensor. In analyzing the behavior of the beam, the Galerkin method was used to discretize two coupled partial-differential equations (PDE), namely the linear beam deflection equation and the Reynolds squeeze-film damping equation. Abdel-Rahman et al. [11] constructed a nonlinear model of an electrically-actuated micro-beam and determined the maximum stable range of travel using a numerical shooting method. Chen et al. [12] used a hybrid differential transformation/finite difference method to analyze the nonlinear dynamic behavior of a clamped–clamped micro-beam in the absence of a squeeze-film damping effect. The same group then used the same method to analyze the nonlinear dynamic response of an electrostatically-actuated micro-circular plate subject to both residual stress and a uniform hydrostatic pressure on the upper surface [13].

To increase the actuation efficiency and detection sensitivity of MEMS-based devices, the separation distance between the two electrodes must be minimized and the overlapping area between them maximized. Under such conditions, a squeeze-film damping effect occurs between the two electrodes as the upper electrode deforms [14]. Squeeze-film damping can be modeled using the Reynolds equation, which is derived from the Navier–Stokes equations and the continuity equation. The squeeze-film damping effect can be described by a continuum model based on the Reynolds equation provided that the Knusden number [25] has a sufficiently low value (i.e., $K_n < 0.01$). When analyzing the dynamic behavior of MEMS-based devices, it is essential that the effects of squeeze-film damping are taken into account. Krylov and Maimon [15] examined the transient dynamic behavior of a micro-beam subject to electrostatic forces, squeeze-film damping, and rotational inertia. Seidel et al. [16] developed a capacitive parallel-plate micro-accelerometer based on a harmonic oscillator with a constant damping coefficient. The experimental and numerical results showed that the resonance frequency increased with an increasing residual pressure in the air gap. Younis [17] presented a hybrid numerical/analytical method for simulating MEMS systems subject to squeeze-film damping in multi-physics fields.

Various researchers have investigated the behavior of micro-beams under a persistent AC voltage [18] or a combined DC/AC voltage [19]. In the latter case, the DC component offsets the beam to a new equilibrium position, while the AC component drives the beam periodically around this newly-established position. Younis et al. [20] showed that the application of an AC load with an appropriate magnitude and frequency improves the response speed of membrane-based MEMs devices and reduces the DC pull-in voltage. Younis [21] examined the dynamic behavior of a micro-beam subject to combined DC/AC loading and developed an analytical expression for approximating the micro-beam motion under resonance conditions.

Differential transformation (DT) theory was first proposed by Zhao in 1986 as a means of solving linear and nonlinear initial value problems in the circuit analysis field. However, in more recent times, researchers have combined the DT method with a finite difference (FD) approximation scheme in order to investigate a wide variety of mechanical and microfluidic phenomena. For example, Liu et al. [22] used a hybrid DT and FD method to analyze the nonlinear dynamic behavior of cantilever micro-beams subject to electrostatic actuation. Chen et al. [23] employed a hybrid DT and FD method to investigate the entropy generated within a mixed convection flow with viscous dissipation effects in a parallel-plate vertical channel. Kuo and Chen [24] used the hybrid DT and FD method to solve the nonlinear Burgers' equation for various values of the Reynolds number.

The present study investigates the dynamic response of an electrically-actuated clamped–clamped micro-beam subject to squeeze-film damping. An analytical model of the micro-beam behavior is proposed based on a nonlinear deflection equation and a nonlinear Reynolds equation. The model is solved under various DC and DC/AC actuating conditions using the hybrid DT and FD method. Moreover, the actuation conditions which ensure the stability of the micro-beam are determined by means of phase portraits. The remainder of this paper is organized as follows. Section 2 derives analytical models for the deflection of the clamped–clamped micro-beam and the squeeze-film damping effect, respectively, and then describes the use of the hybrid DT and FD method in solving the coupled governing equation. Section 3 investigates the dynamic response of the micro-beam under pure DC actuation conditions and combined DC/AC actuation conditions, respectively. In addition, the effects of the AC voltage, excitation frequency, residual stress, and ambient pressure on the displacement of the micro-beam are systematically explored. Section 4 provides some brief concluding remarks and indicates the intended direction of future research. In the proposed approach, each term in the normalized nonlinear governing equation of the clamped–clamped micro-beam is processed using the DT method with respect to the time domain t^*. The transformed equation is then processed using the FD approximation method with respect to the position domain x^*. Compared to existing methods such as the finite difference method, the hybrid numerical scheme has the advantages of an explicit physical meaning and a greater computational speed and simplicity.