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Chaos control of fractional order Rabinovich–Fabrikant system and synchronization between chaotic and chaos controlled fractional order Rabinovich–Fabrikant system

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ABSTRACT

In this article the local stability of the Rabinovich–Fabrikant (R–F) chaotic system with fractional order time derivative is analyzed using fractional Routh–Hurwitz stability criterion. Feedback control method is used to control chaos in the considered fractional order system and after controlling the chaos the authors have introduced the synchronization between fractional order non-chaotic R–F system and the chaotic R–F system at various equilibrium points. The fractional derivative is described in the Caputo sense. Numerical simulation results which are carried out using Adams–Boshforth–Moulton method show that the method is effective and reliable for synchronizing the systems.

1. Introduction

A wide class of physical phenomena can be described by mathematical models. Simple nonlinear dynamical system and even piecewise linear system can exhibit complete unpredictable behavior known as chaos, which is an active area of research to the scientific community working in nonlinear sciences, through this has no unified and universal definitions in the scientific literature. It is described by the set of mathematical equations containing both dynamic and static variables. Chaotic system is bounded nonlinear deterministic system which has a periodic long-term nature depending on initial condition. Again due to sensitive dependence of chaotic dynamics on initial conditions [1], there is always possibility of exponential spreading of trajectories of the systems emerging from initial conditions during coupling of the systems. Dynamic of chaos has a very interesting nonlinear effect, which had been intensively studied by Lorenz [2], who found the first canonical chaotic attractor in the year 1963.

In the last few decades, fractional order modeling has been an active field of research both from the theoretical and the applied perspectives since they are naturally related to the systems with memory which prevails for most of the physical and scientific system. Fractional calculus is a field of applied mathematics that deals with derivatives and integrals of arbitrary orders and have applications in various fields of science and engineering ([3–7]). Fractional differential equations which are generalizations of classical differential equations describe the memory effect, and it is the major advantage over integer-order derivatives [8]. The chaotic dynamics of fractional order systems is an important topic of study in nonlinear dynamics. In the last few years this area of research has been growing rapidly [9].

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In the present article the authors have used a simple algorithm that allows the control laws that stabilizes the chaotic systems around the unstable equilibrium points and synchronized the chaotic system. Since the chaotic systems are very sensitive to initial conditions, so for this reason, chaotic systems are difficult to control and synchronized. Recently, controlling the chaos and synchronization of chaotic systems of complex dynamical systems has attracted researchers in the field of engineering and science. Research efforts have investigated the chaos control in many physical chaotic systems ([10,11]). Synchronization is a phenomenon occurs when two or more nonlinear systems are coupled, which is nowadays very active area of research in nonlinear dynamics. Actually synchronization is caused due to transformation of dynamical variables of two chaotic systems viz., drive (master) and response (slave) systems. The study of synchronization is extremely needed so that the trajectories of two systems will converge and they will remain in step with each other to do the structurally stable coupling.

The pioneering works of Ott et al. [12] and Pecora and Carroll [13] introduced a method about chaos control of chaotic system and synchronization between two identical or non identical systems has attracted a great deal of interest in various fields and engineering ([14–17]). In recent years, many researchers and engineers have devoted their efforts to chaos control and synchronization, including stabilization of unstable equilibrium points [18]. There are various schemes to achieve chaos control and synchronization, such as linear and nonlinear feedback control, adaptive control, active control, sliding mode control etc. ([19–22]). Among two main approaches for controlling chaos feedback control and non-feedback control, the first one is especially attractive and has been commonly applied to practical implementation due to its simplicity in configuration. In 1997, Bai and Lonngren [23] have proposed Active control method, which has received a lot of attention to the researchers working in the area of nonlinear dynamics due to its simplicity and easy to implement in applications of synchronization and anti- synchronization of coupling of a pair of chaotic for both standard order and fractional order cases ([24–29]).

The Rabinovich–Fabrikant (R–F) equations introduced by Mikhail Rabinovich and Anatoly Fabrikant [30] are a set of three coupled ordinary differential equations. For certain values of parameters the system is chaotic and for others it tends to a stable periodic orbit. Danca and Chen [31] predicted that due to the presence of quadratic and cubic terms, it is hard to analyze the system. The stochastic nature of the system arising from the modulation instability is found in non-equilibrium dissipative medium [32].

In the present article the authors have studied the dynamical behavior, chaos control and synchronization of fractional order R–F system. It is found that the chaotic attractor exists in the fractional-order R–F system. Furthermore, fractional Routh–Hurwitz conditions [33] are used to study the stability conditions in the fractional-order R–F system and the conditions for linear feedback control have been obtained for controlling chaos in the considered system. The authors have used the active control method for synchronization between fractional order chaotic and chaos controlled R–F system. Using the Adams–Boshforth–Moulton method ([34,35]), numerical simulation is carried out for different particular cases.

2. Some preliminaries and stability condition

2.1. Fractional calculus

Definition 1. A real function $f(t)$, $t > 0$, is said to be in the space C_μ, $\mu \in \Re$, if there exists a real number $p > \mu$, such that $f(t) = t^p f_1(t)$, where $f_1(t) \in C(0, \infty)$, and it is said to be in the space C_μ^n if and only if $f^{(n)} \in C_\mu$, $n \in N$.

Definition 2. The Riemann–Liouville fractional integral operator (J_t^α) of order $\alpha > 0$, of a function $f \in C_\mu$, $\mu \geqslant -1$, is defined as

$$J_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\xi)^{\alpha-1} f(\xi)\, d\xi, \quad \alpha > 0, \quad t > 0, \tag{1}$$

where $\Gamma(.)$ is the well-known gamma function.

Definition 3. The fractional derivative D_t^α of $f(t)$, in the Caputo sense is defined as

$$D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)} \int_0^t (t-\xi)^{n-\alpha-1} f^{(n)}(\xi)\, d\xi, \tag{2}$$

for $n-1 < \alpha < n,\ n \in N,\ t > 0,\ f \in C_{-1}^n$.

The important reason of choosing Caputo derivatives for solving initial value fractional order differential equations is that the Caputo derivative of a constant is zero, whereas the Riemann–Liouville fractional derivative of constant is not equal to zero.

2.2. Stability of the system

Consider a three-dimensional fractional order system