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A versatile mathematical approach for environmental geomechanic modelling based on stress state decomposition

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A B S T R A C T

The main goal of the present paper is to present a mathematical framework for modelling multi-phase non-saturated soil consolidation with pollutant transport based on stress state configurations with special emphasis in its versatility. Non-linear saturation and permeability dependence on suction for both water and pollutant transport is regarded. Furthermore, through the introduction of a suction saturation surface instead of simple suction saturation curves, the implementation of the saturation–suction coupling effect is considerably simplified. The achieved differential equation system is discretized within a Galerkin approach along with the finite element method implementation. A widespread set of practical situations is encompassed by simply setting certain coefficients of the discrete system of equation according to concrete problem conditions. When the model is coped with certain selected fringe conditions, the approach adaptability feature came up showing a robust performance.

1. Introduction

Mathematical models for solving porous media consolidation stand for an issue widely studied in modern computational mechanics. From the pioneer Biot's work [1] to nowadays complex and robust approaches, a broad range of mechanical situations, thermal conditions, transported fluids, boundary conditions and load types were regarded. The two-phase and three-phase non-saturated cases were undertaken by Ai et al. [2] and Lewis and Schrefler [3] respectively. The non isothermal analysis in saturated models was included in Masters et al. [4], whereas in Yang et al. [5] the non-isothermal case was extended to the unsaturated situation.

Regarding environmental geo-mechanics, different authors have taken on this issue from several standpoints with different aspects or hypothesis under consideration. In Li and Zienkiewicz [6], the fluid transport through porous media in one or two phases separated by an interface with no chemical reactions or components interchange between the phases was presented. In Khoei and Haghighat [7] a solution for material interfaces was brought about independently of element boundaries with an enhanced finite element method (FEM). In Schrefler [8], a mathematical framework assuming a multiphase porous system with voids filled with water, water vapor, dry air and pollutant, relying on Hassanizadeh and Gray averaging theories, see Refs. [9–11], was addressed. Therein and for non isothermal flux, see Khalili and Loret [12], the thermodynamics

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properties of the interface between the different porous media constituents were regarded (Lewis and Schrefler [3]) and, in addition, the relationship between the interface surface tension and the capillary pressure is carefully analyzed.

Furthermore, in Dasgupta et al. [13] the finite difference method was selected for solving a problem of groundwater contamination in a waste-disposal area in Miami, Florida. The relative permeability issue, arising in when wetting and non-wetting phases are regarded, was addressed in Noaman and El-Khatib [14].

Mroginski et al. [15] described a kind of odd relationship that bonds the vertical displacements and the degree of pollutant saturation.

Different degrees of heterogeneity were considered by Ai et al. [16] as well as by Su et al. [17]. In the former, a horizontal multilayered soil with anisotropic permeability undergoing square load was analyzed whilst in the later a first order homogenization on the Representative Volume Element (RVE) was adopted to overcome the heterogeneous issue. In Royer and Boutin [18], the physical interpretation of the three characteristic behaviors of homogenized dual-porosity is evaluated along with memory effects.

Regarding to the numerical solution of the boundary value problem in deformable porous media, different approaches were presented. The coupled consolidation problem of multiphasic porous media was discussed in [6,8,15]. Mroginski and Etse [19] propose a finite element (FE) formulation with C1-continuous interpolation functions for non-local saturated porous media. The reference [3] presents a deep study of the numerical solution of coupled consolidation problems in partially saturated soil within the FE framework.

When damage on soil structure is under consideration, [20] assess the consolidation response of a saturated porous media in which structural damage is induced.

Concerning with the constitutive model, Alonso et al. [21] carried out a clay-type partially saturated soils development within the framework of hardening plasticity using two independent sets of stress variables: the excess of total stress over air pressure and the suction. A model based on suction controlled triaxial tests was presented by Sun et al. [22], whereas Graziano and Lancellotta [23] dealt with the derivation of an evolution constitutive equation for deformable porous media.

Khalili et al. [24] presented a mathematical approach for isothermal partially saturated media grounded in a stress state decomposition though regardless of the saturation or the induced matric suction coupling effect. For this issue was subject of large controversial, in Di Rado et al. [25] the evidence of the highly non linear effect that saturation–suction coupling effect renders to the constitutive model and its influence on the symmetry loss in the main system of equations for the isothermal case, were properly settled down. Moreover, in the present manuscript, two advances are brought into consideration with respect to ref [25]: the addition of an immiscible pollutant phase and the introduction of a surface of saturation–suction relationships accounting for the different fluid content instead using a curve for the mixture of fluids. Additionally, a generalization for N immiscible fluid phases is outlined.

This improved mathematical approach, besides spanning a vast sort of isothermal consolidation problems, inherits the three phase model's ductility (see Di Rado et al. [25]) allowing a straightforward reduction to some others problems with a more easy-solving form, namely, the saturated case, non-saturated without suction coupling, consolidation with no presence of pollutant, consolidation with presence of pollutant but without suction coupling, etc. The aforementioned reduction may be carried out by merely setting some coefficients to the required value.

With respect to the mechanical behavior of the soil skeleton (i.e. the constitutive tensor), the same restrictions stated in Ref. [25] hold. One last issue must be underscored. The non-saturated soil consolidation analysis without thermal effects is of great interest for civil constructions like buildings and earth dams, especially when the location area is placed in the north east region of Argentine or south of Paraguay and Brazil. In these locations, many important cities are situated in ancient river's valleys where clay, lime-clay or even sandy soil type with degrees of saturation over 70% (generally due to the groundwater table position) are commonly found. Along with the classical clay-type soil consolidation problem, a brand new situation arises due to the allocation of new industries: the environmental damage. This facts and the possibility of extending the previously developed code were the principal motivations for the present work.

2. The governing equations

2.1. Introduction

For classical mechanic analyses, a continuum distribution of existent particles, either fluid or solid for which the balance laws and constitutive relationships are valid, is frequently accepted. For the case in point, there is an omnipresent phase, i.e., the solid one or solid skeleton, whose voids are taken to be filled with fluid (gas or liquid) separated by a membrane called interface. The difference between the constituents and phases should be emphasized here. On the one hand, the phases are chemically homogeneous portions of the multiphase system which mechanical behavior is assumed to be uniform. On the other hand, the constituents are the individual parts that give rise to the different phases but acting each one independently.

There are two possible levels for describing the multiphase media intergranular configuration: The macroscopic and the microscopic level. At a microscopic level, the actual porous media structure is regarded (see Fig. 1(a)) and, due to this situation, the governing equations are established considering each constituent separately giving rise to a complicated solution, let alone the assessing of microscopic physical and chemical properties. Considering the aforementioned reasons and adding that the microscopic description is generally beyond the civil engineering goals (see Ref. [3]), the macroscopic description is