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An analytical transfer function method to solve inverse heat conduction problems

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ABSTRACT

This work proposes a transfer function identification (or impulse response) method to solve inverse heat conduction problems. The technique is based on Green's function and the equivalence between thermal and dynamic systems. The inverse heat conduction problems, 1D and 3D transient named $X22$ and $X33Y33Z33$, respectively, are selected to present the fundamentals of the method proposed. The 1D-transient case is a classic heat conduction problem used to obtain thermophysical properties and the 3D-transient problem studied describes a machining process. From the temperature profile (hypothetical or experimental temperature far from the heat source) and knowing the transfer function it is possible to estimate the heat flux by different approaches: deconvolution, spectral densities estimation or inverse fast Fourier procedure. MATLAB codes were used. The work is concluded with the application of the technique in an experimental case of temperature estimation at the tool-work-piece interface during a machining process.

1. Introduction

Inverse problems have important applications in many areas, with special emphasis on engineering and medicine and can be applied in various ways. The main feature of this type of approach is to obtain the solution of the physical problem indirectly, for example, determination of thermal fields at surfaces without access, obtaining the force applied to a complex structure from knowledge of the response and transfer function which describes the system, or the diagnosis of a disease by computerized tomography. In all cases, the boundary conditions of these problems are not known or of difficult access. The problem can be solved using information from sensors located at accessible points.

In direct problems of heat conduction, if the heat flux (the cause) is known, the temperature field (the effect) can then be determined. Whereas for an inverse problem the heat flux is estimated from knowledge of the temperature at a location of easy access. Thus experimental temperatures can be used to obtain: thermal properties, surface heat flux, an internal heat source or the temperature at a surface without direct access, among others.

The main characteristic of the inverse heat conduction problem (IHCP) versus a well-posed direct heat conduction problem is that it leads to solutions that generally are not unique or stable to small changes in the given data [1].

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In literature, a variety of analytical and numerical approaches are proposed for the solution of inverse problems in heat conduction. Based on the least squares method and Duhamel theorem, Beck et al. [2] developed the specified sequential function method, which is still one of the techniques most used for solving inverse problems. The technique consists of successive minimization of the error estimated for only the current time and some future time steps. The function specification method has the two advantages that (a) it is simples in concept and (b) it does not change the physics of the problem, since the intrinsic parabolic nature of the problem is unchanged. Some researchers have proposed adaptations of this method seeking minimization of problems arising from the existence of measurement errors. The objective is to obtain greater stability of the algorithm as proposed by Keanini et al. [3]. In this case, Keanini et al. [3] proposed a modified method of the specified sequential function to stabilize the solution for an inverse parabolic problem of heat conduction. The method uses computational time steps which are larger than the small experimental sampling intervals as well as future time steps that are equal to the sampling intervals.

The IHCP is difficult because it is extremely sensitive to measurement errors. Other important effects can be the presence of the lag and damping on experimental data. Another problem that appears in IHCP is related to the time sample. For example, the use of small times steps frequently introduces instabilities in the solution of the IHCP [4]. It can be observed that the conditions of small time steps have the opposite effect in the IHCP compared to that in the numerical solution of the heat conduction equation.

Tikhonov regularization and iterative regularization [5] are usually presented as whole domain methods in which all the heat flux components are simultaneously estimated for all times (and position, if multidimensional). Two advantages of these methods are that they have had rigorous mathematical investigation and can be applied very generally.

Another technique that uses regularization procedure is the conjugate gradient method with adjoint equation described in detail in Alifanov [1] or Özişik and Orlande [6]. This technique is based on an optimization process using iterative regularization, that is, the results of minimization of the objective function tend to stabilize in function of the number of iterations. This methodology can be employed for the solution of linear and non-linear inverse problems, as well as problems of parameter estimation.

Lesnic et al. [7] present another way to solve IHCP. The procedure is to introduce the least squares, regularization and energy method into the boundary element method (BEM) formulation. The discretization of the IHCP performed by using the BEM has the advantage that no domain discretization is need as requested when using finite differences or finite elements. In this work, a solution of the one-dimensional, linear, inverse, unsteady heat conduction is a slab geometry is analyzed. Temperature measurements in time are taken with a sensor positioned at an arbitrary location within the solid material.

There are several numerical optimization tools used for the solution of inverse problems. The golden section is one of the most popular techniques for the estimation of maximum, minimum or zero of functions of only one variable [8].

It is possible to analyze heat conduction problems by making analogy to dynamic systems. In dynamic systems three variables are studied: the excitement, the transfer function and the system response. The problems are solved knowing two variables and estimating the third. In the case of inverse problems, from the knowledge of the transfer function the system and its response (effect), the excitation (cause) is estimated. In this sense, more recently filters have been employed for solving inverse heat transfer problems, such as Kalman filters and dynamic state observers. Blum and Marquardt [9] presented a solution for an inverse heat transfer problem based on dynamic observers. The inverse problem was interpreted as a low pass filter of real components of the real signal while rejecting the high frequency components to avoid excessive amplification of the noise effect on estimation. The algorithm showed good results in one-dimensional problems.

Hensel [10] has discussed inverse solution techniques using analytical and transfer function. For example, he has presented a temperature analytical solution for the direct problem as a function of an arbitrary transient heat flux (first appeared in a report [11]). This solution has been obtained by using Laplace and Fourier transforms. This solution is then used to generate simulated temperature measurement data at several interior points considering various types of heat flux shuch as, for example, linear periodic square wave surface flux, linear triangular wave or linear impulse heat flux. After the inverse procedure, taking the first derivative of the direct solution with respect to x, the heat flux at any position in the solid can be evaluated. Hensel [10] presents an inverse heat procedure for 1D case using a frequency domain adjoint algorithm. He takes the Fourier transform of the governing equation and also takes two discrete internal measurements. In this case, the inverse problem is solved numerically at each frequency by using the finite difference method. He argues that the finite difference adjoint is analogous to the analytical problem and has great advantage of to be extended to multi-dimensional problems. It can be mentioned here that analytical solution is used to obtain temperature measurement simulated and the direct solution problem. The inverse technique uses space marching algorithm. Contrary to the method proposed here, that technique is limited to interior data measurements and the analytical solution, as well the transfer function, are used only for obtaining the solution of the direct problem.

Regarding the techniques presented, the specified function algorithm is of easy implementation and low computational cost. However, it has not good stability when the presence of experimental noise is of large proportion [2] suffering the influence of local minima. As mentioned, a solution for minimizing these problems is to implement regularization techniques [2]. Another disadvantage of this algorithm is the high mathematical complexity of implementation when estimating heat flux components with spatial and temporal variation. The technique of conjugate gradient with adjoint equation also shows instabilities in the vicinity of local minima.