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Dynamic reconstruction algorithm for electrical capacitance tomography based on the proper orthogonal decomposition

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ABSTRACT

Due to the vivid visualizations obtained of the spatial material distributions of inaccessible objects, electrical capacitance tomography (ECT) is considered to be a promising method for the monitoring and control of various industrial processes in which image reconstruction algorithms play important roles in practical applications. In this study, the proper orthogonal decomposition (POD) method is used to derive a low-dimensional model for ECT imaging problems. We propose a POD-based dimensionality reduction dynamic imaging model, which incorporates the time-varying properties of dynamic imaging objects and prior knowledge obtained from previous measurements, other sensors, or numerical simulation results to simultaneously improve the accuracy and speed of image reconstruction. In the framework of this POD-based low-dimensional imaging model, we propose a new objective functional that integrates additional prior information related to imaging objects to convert the ECT image reconstruction task into an optimization problem. The split Bregman iteration (SBI) method is employed to search for the optimal solution to the proposed objective functional. Unlike standard pixel-based imaging methods, the proposed low-dimensional imaging model is obtained by projecting the original unknown variables onto subspaces spanned by a set of orthogonal basis vectors, where the unknown images are reconstructed indirectly by estimating a low-dimensional coefficient vector. Our theoretical study and numerical simulation results validate the superior performance of the proposed imaging method in alleviating the ill-posedness of the ECT image reconstruction problem, as well as increasing the imaging quality, decreasing the computational cost, improving the reconstruction speed, and enhancing robustness.

1. Introduction

Electrical capacitance tomography (ECT) is used to acquire time-varying spatial material distribution information for inaccessible objects during the monitoring of the multiphase flow systems, chemical reactors, pneumatic conveying systems, and combustion processes, thereby improving the efficiency, safety, and reliability of systems, as well as reducing energy consumption and pollutant emissions [1–10]. As a visualization measurement approach, the ECT method has become an

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active research field, where great progress has been made in practical applications [1–10] and image reconstruction algorithms [11–36].

It is beneficial to reconstruct high-quality images rapidly in practical applications. In recent decades, the critical issue of improving the quality of imaging has attracted much attention and various algorithms had been proposed for ECT image reconstruction, including static imaging algorithms [11–30] and dynamic imaging methods [31–36]. Static imaging methods have been studied intensively and numerous advances have been reported. It should be mentioned that static imaging methods are often employed to implement dynamic imaging tasks. However, because they fail to utilize the time-varying properties of dynamic imaging objects, static imaging methods often yield noticeable reconstruction artifacts. Given the increasing requirement for the imaging of time-varying objects, dynamic imaging algorithms have attracted growing attention in recent years. In general, developing fast and stable numerical methods for solving the dynamic imaging problem still remains a challenging task.

Fast imaging plays a crucial role in capturing the time-varying behaviors of imaging objects to help understand their underlying physical or chemical mechanisms. Popular approaches for accelerating the imaging speed include reducing the computational cost and improving the computational capacity of computers, although the latter method may be impractical for ECT applications due to the high costs involved. Methods for reducing the computational cost employ two approaches: decreasing the computational cost by designing a suitable algorithmic structure and reducing the dimensionality of the unknown variables. For the ECT imaging problem, reducing the dimensionality of the unknown variables is a promising approach for improving the imaging speed. In particular, one of the major challenges in standard imaging algorithms is that the number of unknown variables greatly exceeds that in the independent equations, which makes the imaging problem underdetermined and ill-posed. In order to alleviate this critical problem, a natural approach is to increase the number of measurement data. Unfortunately, increasing the number of measurement data is not a trivial task due to the limits on practical measurement conditions. Obviously, it is crucial to develop new approaches to address this critical problem.

Given the discussion above, it is natural to ask whether it is possible to improve the reconstruction quality and the imaging speed simultaneously? If the answer is yes, another critical problem is how to achieve this goal? If the unknown variables can be represented by a low-dimensional vector, the imaging speed can be improved and the underdetermined attribute and the ill-posed nature of the ECT imaging model can be alleviated. The proper orthogonal decomposition (POD) approach is a popular dimensionality reduction method, which has been successful in numerous applications in various fields. The essence of the POD method depends on projecting the unknown variables onto the subspaces spanned by the set of the orthogonal basis vectors to obtain a low-dimensional model. For the ECT imaging problem, if the original unknown variables can be represented by a POD-based low-dimensional vector, the images can be reconstructed indirectly by estimating the low-dimensional POD coefficients, thereby reducing the computational cost, alleviating the underdetermined property and the ill-posed nature, and improving the imaging speed, which are highly beneficial for ECT imaging tasks. In this study, we aim to develop a POD-based imaging method that improves the reconstruction speed and the imaging quality simultaneously. The main contributions of this study can be summarized as follows.

(1) The original unknown variables are projected onto the subspaces spanned by the set of orthogonal basis vectors to obtain a low-dimensional model, where the images are reconstructed indirectly by estimating a POD-based low-dimensional coefficient vector. It should be noted that reducing the dimensionality of unknown variables will alleviate the underdetermined property and the ill-posed nature, decrease the computational cost, improve the reconstruction speed, and enhance robustness.
(2) In previous studies, popular approaches for improving the imaging speed involved elaborate algorithmic designs. However, this approach cannot overcome the bottleneck that restricts imaging speed improvement because the number of unknown variables remains unchanged. In this study, we improve the imaging speed by POD-based dimensionality reduction of the unknown variables.
(3) Unlike standard dynamic imaging methods, the basis vectors that we use for dimensionality reduction are derived from previous measurements, other sensors, or numerical simulation results, and thus the prior knowledge related to the expected solutions is increased because the basis vectors are extracted from the physical properties of the system of interest and they contain the main features of the expected solutions.
(4) Inaccurate input data restricts imaging quality improvements. In this study, we use the M-estimation method and dimensionality reduction of the unknown variables to improve the robustness of the estimation and to decrease the sensitivity of the final inversion solution to ubiquitous noise.
(5) By incorporating ECT measurement information, the time-varying behaviors of the measured objects, and dimensionality reduction, we propose a POD-based low-dimensional dynamic imaging model. We also propose an objective functional that utilizes prior knowledge associated with the time-varying properties of a dynamic imaging object to convert the ECT imaging task into an optimization problem by introducing the framework of the Tikhonov regularization method.
(6) Our theoretical research and numerical simulation results demonstrate the superior performance of the proposed dimensionality reduction-based dynamic imaging method in alleviating the ill-posedness of the ECT imaging problem, increasing the imaging quality, decreasing the computational cost, improving the reconstruction speed, and enhancing robustness.