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## Semi-analytical approach for analyzing the nonlinear dynamic torsional buckling of stiffened functionally graded material circular cylindrical shells surrounded by an elastic medium



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### ABSTRACT

In this study, we propose a semi-analytical method for analyzing the nonlinear dynamic behavior of functionally graded material (FGM) thin circular cylindrical shells surrounded by an elastic medium under a time-dependent torsional load. The shell is stiffened by a closely spaced orthogonal FGM stiffeners system, which is attached to the inner surface of the shell. The elastic medium is assumed to follow the two-parameter elastic foundation model proposed by Pasternak. The fundamental relations and governing equations for stiffened cylindrical shells are derived using the Donnell shell theory with von Karman geometrical nonlinearity and the smeared stiffeners technique. A deflection function with three terms is employed to consider the nonlinear buckling shape in a more correct manner. The Galerkin method is used. The critical dynamic torsional load is found by using the fourth order Runge–Kutta method and Budiansky–Roth criterion. Based on computations, we investigated the effects of the stiffener, foundations, material, and dimensional parameters on the dynamic buckling behavior of the shell.

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## 1. Introduction

Circular cylindrical shells are employed widely as elements of various engineering structures such as pressure vessels, pipes, and spacecraft, as well as in other civil, mechanical, and aerospace engineering applications. Most of these shells are acted upon by static and impulsive loads, which cause instability and reduce the strength of the structure. Thus, investigating the nonlinear static and dynamical behaviors of cylindrical shells is an important issue, which has attracted considerable attention from many researchers.

For isotropic and composite shells, many studies have concentrated on buckling and postbuckling analyses, as well as the vibration of the shells. Argento [1] investigated the dynamic stability of a composite circular cylindrical shell subjected to combined axial and torsional loading. The equation of motion for the shell was reduced to a system of Hill equations by means of Fourier series expansions. Using Green's formulation and a perturbation technique, Dasgupta [2] determined the free torsional vibration of a thick isotropic incompressible circular cylindrical shell subjected to uniform external pressure. Using the Galerkin method, Tani and Doki [3] studied the vibration and buckling of fluid-filled and cross-ply laminated orthotropic composite cylindrical shells subjected to torsional load. Sofiyev [4] investigated the torsional buckling of

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cross-ply laminated orthotropic composite cylindrical thin shells under dynamic loading as a power function of time based on the modified Donnell shell theory and Galerkin method. The torsional buckling of elastic cylinders with a hard surface coating layer was studied by Zhang and Fu [5], where the deformations of the core and surface layer were obtained analytically using Navier's equation and a thin shell model, respectively. Xu et al. [6,7] studied the dynamic torsional buckling of cylindrical shells subjected to dynamic torsional loads. A symplectic analytical approach was employed to convert the fundamental equations into the Hamiltonian canonical equations with dual variables. Local buckling of the shell was also discussed in their study. Hui and Du [8] investigated the initial postbuckling behavior of imperfect and antisymmetric cross-ply cylindrical shells under torsion with the reduced-Badorf parameter,  $4 \leq ZH \leq 20$ . Zhang and Han [9] considered the torsional buckling and post-buckling behaviors of imperfect cylindrical shells based on the Karman–Donnell-type shell theory and the singular perturbation method.

Many studies have considered the stability and vibration of shells for functionally graded material (FGM) structures. Batra [10] studied the torsion of cylinders with material moduli that varied only in the axial direction. Wang et al., [11] determined the exact solution and transient behavior for the torsional vibration of a functionally graded finite hollow cylinder. Shen [12] analyzed the buckling shear load and post-buckling equilibrium paths of torsion-loaded FGM shells in thermal environments based on the higher order shear deformation theory (HDST) and singular perturbation method. Huang and Han [13] used an analytical method to investigate the nonlinear buckling behaviors of un-stiffened FGM cylindrical shells (UN-S-FGM) under a static torsional load with the nonlinear large deflection shell theory and Ritz method. Using an analytical method, the dynamic stability analysis of UN-S-FGM cylindrical shells was performed under linearly increasing time-dependent torsional loading based on their linear geometric relationship by Sofiyev and Schnack [14].

For shells resting on elastic foundations, many stability and vibration analyses based on un-stiffened shells have been reported. Sheng and Wang [15] considered the effect of thermal load on the buckling, vibration, and dynamic buckling of UN-S-FGM cylindrical shells embedded in a linear elastic medium by using the first-order shear deformation theory to consider the rotary inertia and transverse shear strains. Shen et al. reported the post-buckling analysis of tensionless Pasternak UN-S-FGM cylindrical shells surrounded by an elastic foundation under a lateral pressure and axial load [16,17] based on the singular perturbation technique and HDST. Using the HDST, Bagherizadeh et al. [18] investigated the mechanical buckling of UN-S-FGM cylindrical shells surrounded by a Pasternak elastic foundation. Sofiyev et al. reported many investigations of the stability and vibration analysis of UN-S-FGM cylindrical and conical shells surrounded by elastic foundations [19–22], where the Galerkin method was employed to determine the buckling load and frequency of the shell.

The stability and vibration are also very interesting issues for eccentrically stiffened FGM (ES-FGM) cylindrical shells. Recently, there have been several studies of the static and dynamic behaviors of plates and shells. For FGM stiffeners, Najafizadeh et al. [23] employed linear stability equations in terms of displacements and reported the buckling results for an FGM cylindrical shell reinforced by FGM rings and stringers under axial compression. With homogeneous stiffeners, Bich et al. [24–26] used an analytical approach to investigate the nonlinear static and dynamic behavior of ES-FGM plates and shallow shells based on the classical shell theory (CST).

By following the direction of FGM stiffeners using the CST with the von Karman geometrical nonlinearity and Galerkin method, Dung and Hoa [27] obtained closed form expressions to determine the critical buckling loads and postbuckling load–deflection curve of ES-FGM circular cylindrical shells under external pressure. Using the same method, Dung and Hoa [28] determined the nonlinear buckling and post-buckling of ES-FGM thin circular cylindrical shells under a static torsional load where the deflection function had three terms.

This review of previous studies shows that the nonlinear dynamic buckling analysis of an ES-FGM cylindrical shell surrounded by an elastic foundation and subjected to dynamic torsional load remains a problem. Thus, the major aim of the present study is to develop an approach to address this issue. A novel aspect of the present study is that the shells reinforced by FGM rings and stringers attached to the inside of shell are subjected to a time-dependent torsional load. The elastic foundation is assumed to follow the two-parameter elastic foundation model proposed by Pasternak [29]. We derive the fundamental relations and governing equations based on the smeared stiffeners technique and the Donnell shell theory with von Karman geometric nonlinearity. The deflection function has three terms, including the linear buckling shape and the nonlinear buckling shape. The Galerkin method is applied to the resulting equations to obtain nonlinear dynamic differential equations. The critical dynamic torsional load is found by using the Runge–Kutta method and Buidiansky–Roth criterion. The numerical convergence method of Runge–Kutta is also employed. We consider the effects of the stiffener, foundation, geometrical parameters, and material properties on the dynamic response of an ES-FGM cylindrical shell. The numerical convergence rate of the method is demonstrated. We performed comparisons with previously reported results to evaluate the accuracy of the proposed method.

## 2. Theoretical formulation

### 2.1. ES-FGM circular cylindrical shells

We consider a thin circular cylindrical shell made of combined ceramic and metallic materials with mean radius  $R$ , thickness  $h$ , and length  $L$ , which is only subjected to a time-dependent torsional load of intensity  $\tau$ . The two butt-ends of the shell are assumed to be deformed only in their planes and they are still circular [30]. The shell coordinate system  $(x, \theta, z)$ ,  $y = R\theta$  is

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