Applied Mathematical Modelling 39 (2015) 7032–7044

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Multidimensional characteristic-based solid boundary condition for incompressible flow calculations

Kamiar Zamzamian [a,*], M.Y. Hashemi [b]

[a] Department of Mechanical Engineering, Tabriz Branch, Islamic Azad University, Tabriz, Iran
[b] Department of Mechanical Engineering, Azarbaijan Shahid Madani University, Tabriz, Iran

ARTICLE INFO

Article history:
Received 28 December 2013
Received in revised form 6 January 2015
Accepted 12 February 2015
Available online 11 March 2015

Keywords:
Artificial compressibility
Incompressible laminar flow
Multidimensional characteristic based scheme (MCB)
Solid boundary condition

ABSTRACT

Using the artificial compressibility (AC) approach for incompressible flows, the Navier–Stokes equations are coupled together to obtain solutions for steady flows, where marching in time methods are applicable. In this study, we present a new method that employs the multidimensional characteristics of AC equations to calculate the solid boundary conditions. The first multidimensional characteristic-based scheme (MCB) for incompressible flows was proposed in a previous study. In the present study, this idea is implemented by using the characteristic structure of AC equations near solid walls together with a ghost cell method to satisfy the solid boundary conditions. We test the new proposed method with two well-known benchmark problems, where the results show that the accuracy and convergence speed of the MCB scheme is improved in many cases.

1. Introduction

During the computation of incompressible flows, it is difficult to decouple the continuity and momentum equations due to the absence of a pressure term in the continuity equation. Thus, many finite volume incompressible flow solvers are present in the pressure-based group. However, in recent years, the artificial compressibility (AC) method proposed by Chorin [1] has been applied used widely to incompressible flow computations. In this method, a term that includes the time derivative of pressure is used to couple the continuity and momentum equations. In previous studies of incompressible flows, different methods have been employed for the discretization of AC equations, including central schemes [2] and upwind schemes [3–6].

Drikakis et al. first proposed the used of the characteristic relation of AC equations to establish a characteristic based (CB) scheme [7], where the AC equations were assumed to be locally one-dimensional in the direction of normal to the cell faces. This method has been used widely to simulate incompressible flows in different conditions, including extensions to three-dimensional flows [8] and multigrid techniques [9], porous media [10], flows with various densities [11,12], turbulent flow simulation [13], and non-Newtonian flows [14]. In addition, the CB scheme was used extensively by Zhao et al. to calculate incompressible flows with energy equations to address two- and three-dimensional problems with unstructured grids in conjunction with parallel computations [15–19].

The traditional CB scheme assumes a local one-dimensional in the direction of normal to the cell interface. Implementing the CB method for two and three dimensions is similar to that based on the assumption of one dimension. Traditionally, the

* Corresponding author.
E-mail address: zamzamian@iaut.ac.ir (K. Zamzamian).

http://dx.doi.org/10.1016/j.apm.2015.02.043
0307-904X/

methods used to calculate the multidimensional nature of flow in compressible flows have been designed to consider many directions for the propagation of information instead of only one direction. Previously, we introduced the first multidimensional characteristic-based scheme (MCB) for incompressible flows [20–22]. This scheme uses more than one direction of wave propagation by employing multidimensional characteristic paths. For further details of this method, please refer to [20–22]. For the solid boundary conditions of incompressible flows, the velocity components on the solid wall are equal to the wall's velocity due to the non-penetration of mass, and thus only the pressure needs to be specified. In all previous studies in this area, ordinary extrapolation from the interior cells in the domain near the wall has been used to estimate the pressure on solid walls. This is the case for incompressible flow simulations in conjunction with the AC method with various numerical characteristic-based schemes [10,14–17,20], or the other schemes discussed above [5,6]. In the present study, we propose a new approach based on the multidimensional characteristic structure of AC equations near the solid walls together with the ghost cell concept for estimating pressure on the solid boundaries.

2. Incompressible flow equations

The non-dimensional two-dimensional integrated form of the Navier–Stokes AC equations is used as follows [11]:

$$\oint_{\Omega} \frac{\partial \mathbf{W}}{\partial t} dV + \oint_{C} (\mathbf{F} dS_x + \mathbf{G} dS_y) = \frac{1}{\mathrm{Re}} \oint_{C} (\mathbf{R} dS_x + \mathbf{S} dS_y), \tag{1}$$

where

$$\mathbf{W} = \begin{bmatrix} p \\ u \\ v \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} \beta u \\ u^2 + p \\ uv \end{bmatrix}, \quad \mathbf{G} = \begin{bmatrix} \beta v \\ vu \\ v^2 + p \end{bmatrix},$$
$$\mathbf{R} = \begin{bmatrix} 0 \\ \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix}, \quad \mathbf{S} = \begin{bmatrix} 0 \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \end{bmatrix},$$

where $\mathbf{W}$ is the flow field variables vector; $\mathbf{F}, \mathbf{G}$ and $\mathbf{R}, \mathbf{S}$ are components of the convective and viscous flux vectors in the x and y directions, respectively, dS_x, dS_y are the components of the surface element vector in the x and y directions, respectively, β is a constant known as the artificial compressibility parameter, and Re is the Reynolds number.

In the discretized form, Eq. (1) reads:

$$A_{i,j} \frac{\partial \mathbf{W}_{i,j}}{\partial t} + \sum_{k=1}^{4} \mathbf{F}_k (\Delta S_x)_k + \sum_{k=1}^{4} \mathbf{G}_k (\Delta S_y)_k = \frac{1}{\mathrm{Re}} \left[\sum_{k=1}^{4} \mathbf{R}_k (\Delta S_x)_k + \sum_{k=1}^{4} \mathbf{S}_k (\Delta S_y)_k \right], \tag{2}$$

where $A_{i,j}$ is the cell area, and for any edge of the cell, $\Delta S_x, \Delta S_y$ can be evaluated as follows.

$$\Delta S_x = \Delta y, \quad \Delta S_y = -\Delta x.$$

3. Two-dimensional characteristic structure for AC equations

The left-hand sides of the AC equations (convective terms) are considered to obtain the characteristic relations, as follows [15]:

$$\begin{aligned} &\frac{\partial p}{\partial t} + \beta \frac{\partial u}{\partial x} + \beta \frac{\partial v}{\partial y} = 0, \\ &\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial p}{\partial x} = 0, \\ &\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + \frac{\partial p}{\partial y} = 0. \end{aligned} \tag{3}$$

A typical characteristic surface obtained by the equation $f(x, y, t) = 0$ is considered to derive the characteristic paths. The kinematics equations on the assumed surface are used to relate the partial derivatives to exact derivatives and then the following relation is obtained [23,24].

$$\begin{bmatrix} \frac{f_t}{\beta} & f_x & f_y \\ f_x & \psi & 0 \\ f_y & 0 & \psi \end{bmatrix} \begin{bmatrix} dp \\ du \\ dv \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{4}$$