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Momentum and heat transfer from a semi-circular cylinder in Bingham plastic fluids

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ABSTRACT

In this study, momentum and heat transfer from a semi-circular cylinder immersed in Bingham plastic fluids have been investigated numerically in the laminar flow regime. The governing differential equations have been solved over wide ranges of conditions as: Reynolds number, $0.1 \leqslant Re \leqslant 30$, Prandtl number, $1 \leqslant Pr \leqslant 100$ and Bingham number, $0 \leqslant Bn \leqslant 10^3$. New extensive results on the size and shape of the yielded/unyielded zones, drag coefficient and Nusselt number are presented and discussed. The detailed flow and temperature fields in the vicinity of the cylinder surface are examined in terms of the streamline and isotherm contours respectively. Next, the influence of the type of thermal boundary condition like constant wall temperature (CWT) and constant heat flux (CHF) imposed on the surface of the semi-cylinder has been also discussed. Irrespective of the type of the thermal boundary condition prescribed on the surface of the semi-circular cylinder, the functional dependence of the Nusselt number on the governing dimensionless parameters, namely, like Reynolds number, Prandtl number and Bingham number is qualitatively similar. The rate of heat transfer for the constant wall temperature (CWT) is somewhat higher than that for the constant heat flux condition. Finally, the individual and total drag coefficients have been correlated via simple equations in terms of the modified Reynolds number and the average Nusselt number (in terms of the Colburn *j*-factor) as a function of the modified Reynolds number and Prandtl number thereby enabling its prediction in a new application.

1. Introduction

Within the framework of time-independent non-Newtonian fluids [1], the so-called yield stress or visco-plastic fluids have received limited attention in the literature [2–4]. This type of fluid behavior is characterized by the existence of a threshold level of applied stress (yield stress) below which such a substance deforms like an elastic solid and above the yield stress, it deforms like a fluid. While much confusion exists in the literature whether a true yield stress exists or not [3], the steady shear behavior of many structured fluids (foams, emulsions, suspensions, filled polymeric systems, micellar solutions, for instance) is well approximated by postulating the existence of a yield stress. Indeed, this concept is used routinely in pharmaceutical and personal-care product manufacturing sectors, cleaning aids, processed foodstuffs sector to maintained the particles in suspensions for satisfactory end use of such products [5–7]. Apart from such overwhelming pragmatic relevance of the visco-plastic fluids, this class of materials is also of fundamental significance due to their dual behavior, i.e.,

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Nomenclature

Bn	Bingham number, dimensionless
C_D	drag coefficient, dimensionless
C_{DP}	pressure drag coefficient, dimensionless
C	thermal heat capacity of fluid, J kg^{-1} K^{-1}
D	diameter of semi-circular cylinder, m
D_∞	diameter of computational domain, m
F_D	drag force per unit length of the cylinder, N m^{-1}
F_{DP}	pressure component of drag force per unit length of the cylinder, N m^{-1}
h	heat transfer coefficient, W m^{-2} K^{-1}
k	thermal conductivity of fluid, W m^{-1} K^{-1}
m^*	growth rate parameter, dimensionless
n_s	unit normal vector
N	number of elements in the computational domain
N_p	number of grid points on the surface of the semi-circular cylinder
Nu_{avg}	average Nusselt number, dimensionless
Nu	local Nusselt number, dimensionless
p	pressure, dimensionless
Pr	Prandtl number, dimensionless
Pr^*	modified Prandtl number, dimensionless
q_w	heat flux on the surface of the semi-circular cylinder, W m^{-2}
Re	Reynolds number, dimensionless
Re^*	modified Reynolds number, dimensionless
T	temperature of fluid, K
T_w	cylinder surface temperature, K
T_∞	free stream fluid temperature, K
$\mathbf{V}$	velocity vector, dimensionless
V_∞	free stream velocity, m s^{-1}
V_x	velocity component in x-direction, dimensionless
V_y	velocity component in y-direction, dimensionless
x, y	cartesian coordinates, dimensionless

Greek symbols

$\dot{\gamma}$	rate-of-strain tensor, dimensionless
η	apparent viscosity, Pa s
μ_B	plastic viscosity, Pa s
μ_Y	yielding viscosity, Pa s
ρ	density of the fluid, kg m^3
τ	extra stress tensor, dimensionless
τ_0	yield stress, Pa
∇	del operator, m^{-1}

Subscripts

∞	inlet condition
w	cylinder surface condition

Abbreviations

CHF	constant heat flux
CWT	constant wall temperature
PARDISO	parallel direct sparse solver interface

depending upon the prevailing stress levels, a visco-plastic substance exhibits fluid-like or solid like behavior. It is thus conceivable that, in a given configuration, the flow field will be characterized by the coexistence of yielded (fluid-like) and unyielded (solid-like) domains. Such duality of visco-plastic fluids makes their mixing [8,9] and heating/cooling [2,10–12] problematic and challenging. For instance, in the solid-like subregions, heat transfer is severely impeded, as conduction is the main mechanism of heat transfer. Indeed, under certain conditions, this could be the overall rate-limiting step. In spite of such a wide occurrence of visco-plastic fluid behavior, a cursory inspection of the contemporary literature reveals that the bulk of the currently available studies relate to fluid mechanical aspects in duct flows [1,2], mixing vessels [8,9], porous media flows [13,14] and the analogous heat/mass transfer studies are indeed very scarce. In contrast, the corresponding body