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Correlation coefficient of dual hesitant fuzzy sets and its applications

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ABSTRACT

Correlation between two variables or parameters plays a very significant role in statistics. Furthermore, the accuracy in the measurement of the correlation depends upon the data collected for the set of discourse. It is quite evident that in many cases the data collected for various statistical measures are full of uncertainties. A dual hesitant fuzzy set (DHFS) is a generalized form of a hesitant fuzzy set (HFS) and negates the effects of uncertainty inherent in the collected data. In the present paper, the concept of HFS has been replaced with DHFS and the correlation between two DHFSs is obtained. A formula for the correlation coefficient between two DHFSs has been derived. The proposed method was used to determine the coefficient of correlation between different parameters of water in four different lakes in Rajasthan, India.

1. Introduction

The correlation coefficient between any two variables or parameters is used widely in statistics. The Karl Pearson coefficient, proposed in 1895, has been applied to various indices in statistics, such as data analysis and classification, clustering, decision making, medical diagnosis, etc. [1]. Correlation is used to infer the change in a variable correlated with any other variable. It is quite evident that the data collected pertaining to real world problems are fuzzy in nature. To deal with this situation, the concept of correlation has been extended to fuzzy correlation by various researchers, such as Chian and Lin [2], Hang and Hwan [3], Liu and Kao [4] and others.

The concept of Intuitionistic Fuzzy Set (IFS), which is a generalized form of a fuzzy set (FS), has attracted the attention of many researchers, such as Gerstarken and Mako [5], Szmidt et al. [6], Bustince and Burillo [7], Hong and Hwang [8], Hung [9], Hung and Wu [10], Zeng and Li [11], etc. to extend the concept of fuzzy correlation to intuitionistic fuzzy correlation.

Since the introduction of fuzzy sets by Zadeh [12] in 1965, the concept of fuzzy sets has been extended further by various researchers, such as Atanassov's Intuitionistic fuzzy sets [13], fuzzy multisets [14], Type-2 fuzzy sets [15], etc. In 2009 Torra and Narukawa [16] and Torra [17] introduced the concept of HFS. According to Torra [17], the motivation behind the introduction of HFS is that, when defining the membership of an element, the difficulty of establishing the membership degree is not because there is a margin of error (as in IFS) or some possibility distribution (as in type-2 fuzzy sets) on the possible values, but because there is a set of values.

Some aggregation operations and distance measures have also been developed by researchers. By using HFS, Chen et al. [18] derived some formulas for the correlation coefficients and used them to perform clustering analysis for hesitant fuzzy information.

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In 2012, Zhu et al. [19] extended further the concept of HFS to dual hesitant fuzzy set (DHFS). In Atanassov's Intuitionistic Fuzzy Sets (A-IFS), two separate functions handle the membership and non-membership. The membership and non-membership represent the opposite epistemic degrees. Apparently, the membership comes to grip with epistemic certainty and non-membership comes to grip with epistemic uncertainty [19].

In the present paper, a correlation coefficient formula has been derived between two DHFSs. By using concepts from statistics, first, the formulas are developed for the correlation coefficients r_1 and r_2 for membership and non-membership, respectively. The average of r_1 and r_2 determines the coefficient of correlation r between the data represented by two DHFSs.

Furthermore, water is the most precious resource. Both mankind and all aquatic organisms rely on it for their survival. Unfortunately, water has been subjected to massive exploitation and has been severely polluted through anthropogenic activities such as effluent and solid discharge from industries, excessive use of insecticides/ pesticides, disposal of municipal water, etc. The rapid population growth and the resultant urban expansion causes significant pollution (water/air) to be released into the ambient environment [20]. Furthermore, when people dump heated effluent into waterways, this effluent raises water temperature. The increasing water temperature will impact the concentration of dissolved oxygen (DO) and free CO_2 in water, causing an adverse effect on the overall health of a lake. In this study, the ecological systems of four lakes: Jalmahal Lake, Amer Lake, Mevtal Lake and Ramgarh Lake in Rajasthan, India were investigated throughout the year 2011. The three different parameters of lake water observed were: *"Water temperature"*, *"Free CO_2"* and *"Dissolved oxygen (DO)"*. Each were considered for correlation with the other. The change in one may result in change in another. To compute the correlation between these three parameters of lake water, three DHFSs (D_A, D_B and D_C) are assigned to the three linguistic terms namely high temperature, high free CO_2 and high dissolved oxygen, respectively. The formula derived for the correlation coefficient between two DHFSs has been used to evaluate the coefficient of correlation between D_A and D_B, and D_A and D_C.

2. Preliminaries

2.1. Fuzzy sets

For a reference set X, a fuzzy set $\tilde{A}$ can be defined as $\mu_{\tilde{A}}(x) : X \to [0, 1]$, where μ represents the degree of membership of x in $\tilde{A}$ [21].

2.2. Intuitionistic fuzzy sets

An intuitionistic fuzzy set $\tilde{A}^i$ on X is given by $\tilde{A}^i = \{< x, \mu_{\tilde{A}^i}(x), v_{\tilde{A}^i}(x) >\}$, where $\mu_{\tilde{A}^i}(x) : X \to [0, 1]$ and $v_{\tilde{A}^i}(x) : X \to [0, 1]$ such that $0 \leqslant \mu_{\tilde{A}^i}(x) + v_{\tilde{A}^i}(x) \leqslant 1$ for all $x \in X$.

The value $\mu_{\tilde{A}^i}(x)$ is a lower bound on the degree of membership of x that is derived from the evidence for x and $v_{\tilde{A}^i}(x)$ is a lower bound on the negation of x that is derived from the evidence against x.

2.3. Hesitant fuzzy sets

For a reference set X, Torra [16] defined a hesitant fuzzy set H on X in terms of a function $h(x)$ when applied to any $x \in X$ returns a subset of [0, 1].

$H = \{\langle x, h(x) \rangle | x \in X\}$, where $h(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x \in X$ to H and termed as a hesitant fuzzy element (HFE).

Torra [16] defined some special hesitant fuzzy sets as follows:

(i) **Empty set:** $h(x)$ = {0} for all $x \in X$;
(ii) **Full set:** $h(x)$ = {1} for all $x \in X$;
(iii) **Complete ignorance:** $h(x)$ = [0,1] for all $x \in X$;
(iv) **Set for non-sense:** $h(x) = \phi$ for all $x \in X$.

Further, Torra [16] defined HFSs in terms of union of the membership of fuzzy sets. Let $M = \{\mu_1, \mu_2, \dots \mu_N\}$ be a set of N membership functions. Then the HFS associated with M i.e. h_M can be defined as:

$$h_M(x) = \bigcup_{\mu_i \in M} \{\mu_i(x)\}.$$

2.4. Lower and upper bound of HFS

The lower and upper bound of a hesitant fuzzy set H can be defined in terms of HFE $h(x)$ as follows:
Lower bound: $h^-(x) = \min h(x)$;
Upper bound: $h^+(x) = \max h(x)$.