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Onset of triply diffusive convection in a Maxwell fluid saturated porous layer

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ABSTRACT

The linear stability of triply diffusive convection in a binary Maxwell fluid saturated porous layer is investigated. Applying the normal mode method theory, the criterion for the onset of stationary and oscillatory convection is obtained. The modified Darcy–Maxwell model is used as the analysis model, this allows us to make a thorough investigation of the processes of viscoelasticity and diffusions that causes the convection to set in through oscillatory rather than stationary. The effects of Vadasz number, normalized porosity parameter, relaxation parameter, Lewis number and solute Rayleigh number on the system are presented numerically and graphically.

1. Introduction

Double diffusive convection which is driven by the differential diffusion of two properties such as heat and salt are thought to play an important role in many regions, such as the disposal of waste material, groundwater contamination, transport of contaminants in saturated soil, liquid gas storage, food processing and others, some of which are listed by Turner [1]. The excellent reviews of the literature on the applications have been included by Trevisan and Bejan [2] and also documented in the books by Nield and Bejan [3] and Vafai [4].

Although the subject of double-diffusive convection is still an active research area [5–7], however, there are many fluid systems in which more than two components are present. For example, Degens et al. [8] reported that the saline waters of geothermally heated Lake Kivu are strongly stratified by temperature and a salinity which is the sum of comparable concentrations of many salts, while the oceans contain many salts in concentrations less than a few per cent of the sodium chloride concentration. It has been recognized previously that there are important fluid mechanical systems in which the density depends on three or more stratifying agencies having different diffusivities, which can be called multiply diffusive convection [9]. By analogy with the doubly diffusive case in which the density depends on two independently diffusing stratifying agencies, we refer to the isothermal quaternary and non-isothermal ternary (i.e., three-component) cases as being 'triply diffusive'. The subject with more than two components-although more difficult-in the past as nowadays has also attracted the attention of many authors [10–14]. Rionero [12] studied a triply convective–diffusive fluid mixture saturating a porous horizontal layer in the Darcy–Oberbeck–Boussinesq scheme. Tracey [10] developed the linear instability and nonlinear energy stability analyses for the problem of a fluid-saturated porous layer stratified by penetrative thermal convection and two salt

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concentrations. Straughan and Tracey [13] investigated the effect of an internal heat source on the problem of triply-diffusive convection. The multi-component diffusive convection presents a picture of behaviors increasing together with the number of components.

Recently, interest in viscoelastic flows through porous media has grown considerably, due largely to the demands of such diverse fields as biorheology, geophysics, chemical, and petroleum industries. Wang and Tan [16] have made the stability analysis of double diffusive convection in Maxwell fluid in a porous medium. It is worthwhile to point out that the first viscoelastic rate type model, which is still used widely, is due to Maxwell. The onset of double diffusive convection in a viscoelastic saturated porous layer have been considered by many researchers [15–20]. We are unaware, however, of any previous work in the triple-diffusive problem which also introduces penetrative convection in a Maxwell fluid. In this paper, we intend to perform linear stability analysis of a triply diffusive convection in a triply Maxwell fluid saturated porous layer using Darcy–Maxwell model. The Dufour and Soret effects are ignored. The aim of the present paper is to study penetrative convection in a triple-diffusive and how the onset criterion for oscillatory convection is affected by the viscoelastic parameters.

2. Mathematical model

The system is considered that a porous layer of thickness d and infinite horizontal extent bounded blow and above, i.e., $z = 0$ and $z = d$, which is saturated with a Maxwell fluid mixture heated and salted from blow, and the temperature T and concentrations S are held fixed. Consider the Maxwell fluid containing two diffusive properties and let $\kappa_i\ (i = 1, 2)$ is the coefficient of molecular diffusion for the ith component, and S_i is the corresponding concentration. For the purposes of the present paper we consider that positive ΔT and ΔS_i imply the temperature and concentrations are greater at the lower plate. In the Boussinesq approximation the equation of state is:

$$\rho = \rho_0[1 - \alpha(T - T_0) + \beta_1(S_1 - S_{10}) + \beta_2(S_2 - S_{20})] \tag{1}$$

where α, β_1 and β_2 are the thermal and solutal expression coefficients in the medium, T_0, S_{10} and S_{20} are temperature and concentrations at the above plate, respectively.

Considering the vertically downward gravity force $\mathbf{g}$ and neglecting the off-diagonal (Soret, Dufour and cross-diffusion) contributions to the fluxes of the stratifying agencies, by using of the modified Darcy–Maxwell model, the governing system for triply diffusive of Maxwell fluid in a porous layer can be written as follows:

$$\nabla \cdot \mathbf{q} = 0, \tag{2}$$

$$\left(1 + \lambda_1 \frac{\partial}{\partial t}\right)\left(\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{q}}{\partial t} + \nabla \mathbf{p} - \rho \mathbf{g}\right) + \frac{\mu}{K}\mathbf{q} = 0, \tag{3}$$

$$\gamma \frac{\partial T}{\partial t} + \mathbf{q} \cdot \nabla T = \kappa \nabla^2 T, \tag{4}$$

$$\varepsilon \frac{\partial S_1}{\partial t} + \mathbf{q} \cdot \nabla S_1 = \kappa_1 \nabla^2 S_1, \tag{5}$$

$$\varepsilon \frac{\partial S_2}{\partial t} + \mathbf{q} \cdot \nabla S_2 = \kappa_2 \nabla^2 S_2 \tag{6}$$

where $\mathbf{q} = (u, v, w)$ is the Darcy velocity, $\mathbf{p}$ is the pressure, $\mathbf{g}$ is the acceleration due to gravity, μ is the viscosity, λ_1 is the relaxation time, ρ is the density, while K and ε are the permeability and porosity of the medium, $\gamma = (\rho c)_m/(\rho c)_f$ is the ratio of heat capacities, κ and κ_i are effective thermal diffusivity and solutal diffusivities of the medium, respectively.

The basic state of the fluid is assumed to be quiescent. The quantities of the basic state are given by:

$$\mathbf{q}_b = 0, \quad \mathbf{p} = \mathbf{p}_b(z), \quad T = T_b(z), \quad S_1 = S_{1b}(z), \quad S_2 = S_{2b}(z) \tag{7}$$

with the boundary conditions

$$T = T_0 + \Delta T, \quad S_1 = S_{10} + \Delta S_1, \quad S_2 = S_{20} + \Delta S_2 \quad \text{at} \quad z = 0, \tag{8}$$

$$T = T_0, \quad S_1 = S_{10}, \quad S_2 = S_{20} \quad \text{at} \quad z = d. \tag{9}$$

The steady state solutions are given by

$$\begin{aligned} &T_b = T_0 + \Delta T(1 - \frac{z}{d}), \quad S_{1b} = S_{10} + \Delta S_1(1 - \frac{z}{d}), \quad S_{2b} = S_{20} + \Delta S_2(1 - \frac{z}{d}),\\ &\mathbf{p}_b = \mathbf{p}_0 - \rho_0 \mathbf{g} z\left(1 - \alpha\Delta T + \sum_{i=1}^{2}\beta_i \Delta S_i\right) - \rho_0 \mathbf{g} z^2\left(\frac{\alpha \Delta T}{2d} - \frac{1}{2d}\sum_{i=1}^{2}\beta_i \Delta S_i\right). \end{aligned} \tag{10}$$