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Dynamic model for a magnetorheological damper

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ABSTRACT

A lumped mass thermo-mechanical model for the dynamics of a damper filled with a magnetorheological fluid is described, analyzed, and numerically simulated. The model includes friction and temperature effects, and consists of a differential inclusion for the piston displacements coupled with the energy balance equation for the temperature. The fluid viscosity is assumed to be a function the temperature and electrical current, which in practice may be used as the control variable. Numerical simulations of the system behavior are presented. In particular, the simulations of an initial impact show how the subsequent oscillations can be effectively damped.

1. Introduction

In this work we model and numerically simulate the dynamics of a damper that contains a magnetorehological (MR) fluid.[1] Such fluids, typically, consist of micron-size ferrous particles dispersed in a working fluid. They are characterized by rapid changes in the fluid viscosity resulting from the rearrangement of the particles in the fluid due to changes in the applied magnetic field. The application of such fluids in dampers allows control of their damping characteristics, adjusting them to the varying needs of the system. This property qualifies MR fluids as "smart materials." Dampers with MR fluids may offer an improved control of vibrations in airplanes upon landing, in cars, mechanical and medical devices, and industrial machinery. An innovative use may be in vibration control of prosthetic devices.

This work is a continuation of the study in [1], where a basic quasi-static model was developed and numerically simulated. Here, we investigate the dynamic case that takes into account the inertial term and may exhibit oscillations, which the quasi-static model cannot. Our main interests are in the response of the system to an applied periodic force, and to an initial impulse.

A detailed description of the applied aspects of vibration control of systems using MR dampers can be found in the recent monograph [2], where many applications and references can be found. Here, our interest lies in the mathematical and numerical aspects of the model.

The literature on MR fluids is rapidly growing, see, e.g., [3–8] and references therein. Some of the applications described in these references are to semi-active control of the motion of structures caused by seismic disturbances. It is seen that this semi-active control is emerging as one of the first applications of MR fluids. In view of the major earthquake that hit Japan

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[1] MR fluid is abbreviation of magnetorehological fluid.

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(of magnitude 9.0 on the Richter scale, on 11 March 2011) it is a subject of intensive academic and applied study. Moreover, an internet Google search for an MR fluid resulted in a large number of items, mostly industrial but many academic. Additional results are in [9–13] and the references therein. MR dampers were investigated experimentally in [9,10] and modeled as lumped mass systems. Finally, we refer to the item on MR fluids in [14] where additional explanations and references can be found.

The constitutive relations of MR fluids, that is, the relationship between the strain rate and the stress [11–13], and the dependence of the viscosity on the current, which is of fundamental importance, seem to be in their infancy. Related work on smart materials, dealing with constitutive equations for granular materials, can be found in [15–17].

We model the damper as a lumped mass system, as in [1], and propose a general constitutive relation which is represented, essentially, by a viscosity coefficient and a friction bound that depend on the magnetic field via the electric current, and on the system temperature. However, the exact way the viscosity of the MR fluid depends on the current is an open question, and in the theoretical part we do not specify the exact form. This will have to be established experimentally for different MR fluids, possibly by using the model simulations and a parameter identification approach. A dependence of the viscosity coefficient on the temperature can be seen in many fluids, and we assume that this is the case with MR fluids, too. Similarly, the precise dependence of the friction bound on the control variable, the electric current, and on the temperature needs to be determined experimentally. Initial steps in this direction can be found in [9,10] where only the friction bound was assumed to be controlled by the current.

The model consists of a differential inclusion for the displacements of the piston coupled with the energy balance equation for the system temperature. It includes contact and friction which lead to the set inclusion part, and also the restoring force generated by the air bubble. The system is highly nonlinear and its analysis will be the topic of future work. An algorithm for the model was developed in which the contact and friction conditions were regularized. Then, it was written in Mathematica and many examples generated. A few typical ones are presented and discussed in this work.

The rest of the paper is structured as follows. Section 2 presents the setting of the problem and the model. The algorithm for the model is in Section 3, and numerical simulations can be found in Section 4, where four representative examples are presented. Section 5 concludes this work and poses some unresolved issues for future research.

2. Theory/model

We follow [1], with appropriate modifications, and describe a lumped mass model for the dynamics of the thermomechanical state of a damper. The system is depicted in Fig. 1 and consists of a tube filled with an MR fluid, a piston, a piston head, and an air bubble (marked G) which is separated from the MR fluid by a diaphragm. There are thin channels inside the piston head where the MR fluid may flow from one side to the other, and by changing the viscosity of the fluid one can control the fluid flow in these channels.

The heart of the device is the part that allows use of electric current to control the fluid flow in the channels. This is done by a coil, which carries a variable electric current, that is situated inside the piston head, and is connected to a current source via wires in the piston. By changing the electric current in the coil, the magnetic field in the head changes causing changes in the viscosity or damping coefficient ν_D of the fluid inside the channels. In this manner the system's damping is effectively controlled, within certain bounds. Indeed, the purpose of such a 'smart material' damper is to adapt the response of the damper to the rapidly changing system environment.

An external load F, which may be periodic or impulsive, acts on the piston, while the fluid viscosity and the friction between the head and the tube generate opposing forces. The dissipated mechanical energy causes the system temperature to raise, affecting the fluid viscosity. The gas bubble G returns the piston head to its equilibrium position once the load is removed and prevents it from hitting the right end of the tube.

The length of the casing tube is L, and in the absence of loads the MR fluid occupies $0 < x < l$ and the bubble occupies $l < x < L$. The cross section of the tube (and the head) is denoted by A and that of the piston A_p. The combined mass of the piston and the head is m, l_p is the length of the piston shaft, and the static state bubble volume is $V_0 = (L - l)A$.

The model is set in terms of the total or system averaged quantities which depend only on time. We denote by $y = y(t)$ the position of the piston, measured from $x = 0$, and by $\dot{y} = \dot{y}(t)$ its velocity, where a dot above a symbol denotes the time derivative. We denote by $\theta = \theta(t)$ the system temperature, measured with respect to the ambient temperature θ_{amb}. The MR fluid is assumed incompressible and $p = p(t)$ denotes the pressure in the bubble and in the fluid. The resistance force provided by damping is $\nu_D\dot{y}$.

The friction force $\xi = \xi(t)$ which is due to the contact of the piston's head with the casing, is given by

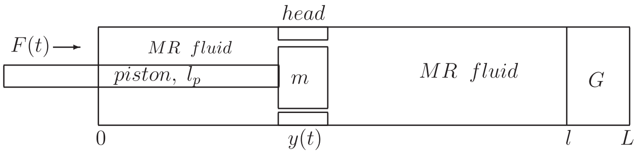

Fig. 1. Schematic MR damper.