Applied Mathematical Modelling 38 (2014) 2377–2397

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A feasible implementation procedure for interval analysis method from measurement data

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ARTICLE INFO

Article history:
Received 4 October 2012
Received in revised form 8 August 2013
Accepted 8 October 2013
Available online 1 November 2013

Keywords:
Uncertainty quantification
Uncertainty propagation
Interval analysis method
Small sample measurement data
Feasible implementation procedure

ABSTRACT

An uncertain quantification and propagation procedure via interval analysis is proposed to deal with the uncertain structural problems in the case of the small sample measurement data in this study. By virtue of the construction of a membership function, a finite number of sample data on uncertain structural parameters are processed, and the effective interval estimation on uncertain parameters can be obtained. Moreover, uncertainty propagation based on interval analysis is performed to obtain the structural responses interval according to the quantified results of the uncertain structural parameters. The proposed method can decrease the demanding on the sample number of measurement data in comparison with the classical probabilistic method. For instance, the former only need several to tens of sample data, whereas the latter usually need several tens to several hundreds of them. The numerical examples illustrate the feasibility and validity of the proposed method for non-probabilistic quantification of limited uncertain information as well as propagation analysis.

1. Introduction

With the continuous development of technology, the complexity of the engineering structural systems gradually increases so that the anticipated influence of the uncertainty on them becomes more and more profound [1]. Traditional methods for dealing with uncertainty engineering problems, namely, probabilistic analysis method need to have sufficient information to determine the probability distributions; however, the experimental data is often limited so that one cannot meet the requirements for the available data to justify the probabilistic reliability model appropriately [2–3]. In recent years, the non-probabilistic convex method of set theory as well as fuzzy theory have not only gained much popularity and even many discussions but also demonstrated a great superiority in processing the uncertainty problems of lack of data information.

Based on the non-probabilistic approach, Ben-Haim, Elishakoff and Qiu [3] proposed and advocated initially to utilize the set-theoretical convex method when dealing with the uncertainty issues. Qiu, Chen and Guo [4–6] applied the above set-theortical convex method to the analysis of structural response, and obtained a respectable results. The set-theoretical convex method can effectively solve the problem of uncertainty with little information, but it also has the defects of interval extension and computation, which may limit the development of this method. In recent ten years, more and more researchers devote to improve the non-probabilistic analysis methods and apply them to engineering problems [7–14].

In the respect of fuzzy theory, since the concept of fuzzy was proposed by Zadeh, the fuzzy method has been extensively applied in the field of information processi, control theory, artificial intelligence and so forth [15–18]. Elishakoff et al. [2]

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http://dx.doi.org/10.1016/j.apm.2013.10.049

introduced the fuzzy sets into the analysis of structural uncertainty, and then the comparison with the probabilistic and convex modeling of uncertainty was derived. Krishnakumar et al. [19] developed a fuzzy logic control scheme that would combine the power of linear dynamic controllers with fuzzy set theory. Ni et al. [20] suggested a method that could forecast the span of the structural fuzzy reliability level based on the fuzzy reliability analysis model; moreover, proposed a procedure for analyzing the fuzzy reliability of structural system when parameters of probabilistic models were fuzzy. Other works of uncertain analysis based on fuzzy theory can be found in Ref. [21–24].

However, most of the studies based on fuzzy theory and set-theoretical convex method are always on the premise of a hypothetical membership function or an assumed interval or ellipsoidal when dealing with uncertain parameters so that the focus of more attention is concentrated on the propagation analysis of uncertainty, while the researches on quantification of initial input parameters, namely according to the limited measurement data to obtain a reasonable interval estimation, so far, are not so many. Schweppe [25] in late 1960s, first proposed an analogous concept based on ellipsoidal modeling, representing the uncertain variables as belonging to an ellipsoid. Elishakoff and Zhu et al. [3,26–28] derived a minimum-volume ellipsoid that encloses the minimum-volume parallelepiped for buckling analysis. Hamle et al. [29] proposed to use the data envelopment analysis to quantify uncertainty. Wang et al. [30] made some exploring researches in regard to the quantification and propagation analysis proceeding from experimental data with the revised minimum volume criterion based on convex modeling method and interval analysis method. In the aspect of technologies of fuzzy quantification, the ranking fuzzy numbers as an important and prerequisite procedure for decision makers plays a leading role [31–33]. Wang and Kerre [34,35] classified the ranking methods into three categories and figured out the rationality of an ordering procedure. The studies of uncertainty quantification based on the probability theory, such as point estimation [36,37], maximum likelihood estimation [38,39], hypothesis testing [40–42], Bayesian method [43–50], etc., have still been drawn attention in recent decades although they generally need a large number of samples from experiments to define the probability density function or determine the statistic characteristics of uncertain parameters.

It is necessary to emphasize that the uncertain analytic methods rely on the way of description on uncertainty, which is always determined by the quantity and the type of available information on uncertainty. In other words, the quantification of uncertain parameters is the basis for other uncertain analyses, including uncertainty propagation, reliability analysis and reliability-based optimization.

In this paper, based on fuzzy estimation method to the expanded uncertainty, a new membership function with reference to limited discrete data will be given, and hence the quantitative description of uncertain parameters can be shown as an estimated interval. In addition, the quantitative results of uncertain parameters will be applied into FE analysis that brings further discussion about structural response. Thus, a complete uncertain analytic process based on practical fuzzy theory from quantification to propagation with primitive measurement data is presented ultimately.

2. Uncertainty quantification based on fuzzy estimation method to the expanded uncertainty

2.1. Processing of interval number

In practical engineering when dealing with the workpiece, in order to get more accurate results, each workpiece would be given multiple measurements. Thus, n interval numbers can be obtained by m workpieces' measurement results as follows.

$$d = (d_1, d_2, \dots, d_i, \dots, d_n) = ([d_{1\min}, d_{1\max}], [d_{2\min}, d_{2\max}], \dots, [d_{i\min}, d_{i\max}], \dots [d_{n\min}, d_{n\max}]) \tag{1}$$

If both the maximum and the minimum value of each workpiece' parameter can be known, then one workpiece forms one interval number, namely $n = m$; and if each workpiece has only one data, the interval number could not be realized as above that a real problem occurs. The solution of this problem is that consider the two adjacent workpieces as a group according to a random sequence or the order of system output, which obtains two data, then sort these two data in an increasing order and thus an interval number forms, namely $n = 0.5m$.

It is easy to understand parameters' measurement results d_i of m workpieces actually constitute an order of interval number. Under the criterion of calculation based on fuzzy-set theory, the definition of mean values and ranges with reference to d_i will be given as follows.

$$\overline{d} = \{\overline{d_i}\} = \{(d_{i\min} + d_{i\max})/2\} \quad i = 1, 2, \dots, n \tag{2}$$

and

$$E_e = \{E_{ei}\} = \{d_{i\max} - d_{i\min}\} \quad i = 1, 2, \dots, n. \tag{3}$$

By fuzzy operation for these n mean values of interval numbers, both fuzzy expectation and dispersion interval about mean value, which used to depict m parameters of workpieces are available, i.e.

$$A = [\overline{A} - \delta_{A1}, \overline{A} + \delta_{A2}] = \overline{A}^{+\delta_{A2}}_{-\delta_{A1}} \tag{4}$$

where A is fuzzy expectation based on mean value, δ_{A1} and δ_{A2} are lower and upper deviations, respectively.