Applied Mathematical Modelling 38 (2014) 2414–2421

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Volterra-type models for nonlinear systems identification

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ARTICLE INFO

Article history:
Received 16 July 2012
Received in revised form 12 September 2013
Accepted 8 October 2013
Available online 1 November 2013

Keywords:
Nonlinear identification
Volterra-type models
Wiener model
Hammerstein model

ABSTRACT

In this work, multi-input multi-output (MIMO) nonlinear process identification is dealt with. In particular, two Volterra-type models are discussed in the context of system identification. These models are: Memory Polynomial (MP) and Modified Generalized Memory Polynomial (MGMP), which can be considered as a generalization of Hammerstein and Wiener models, respectively. Both of them are appealing representations as they allow to describe larger model sets with less parametric complexity. Simulation example is given to illustrate the quality of the obtained models.

1. Introduction

Several model-based nonlinear controller design techniques have been developed in recent decades. Since most of them require models of relatively low complexity, use of first principle models is not an alternative to control design. In this context, empirical modeling is a useful option, because it allows a much greater influence on the model complexity. This influence comes from the fact that before a nonlinear model of a plant is identified, a model structure has to be chosen. Moreover, this model should be capable of representing the dynamical behavior of the plant [1].

The problem of finding a good and simple nonlinear model structure is a complex issue. It is well-known that systems presenting weak nonlinearities could be represented by a Volterra model. In particular, discrete-time systems with fading memory can be approximated arbitrarily well by a discrete-time Volterra model (DTVM), if adequate orders are chosen [2–4].

In general, the most general form of nonlinearity with M-tap memory (i.e., M tap delay lines or memory effects) is described as follows [5]:

$$\hat{y}(t) = \sum_{n=1}^{N} v_M^n(t), \tag{1}$$

with $\hat{y} \in \Re$ and where

$$v_M^n(t) = \sum_{i_1=0}^{M-1} \cdots \sum_{i_n=0}^{M-1} h_n(i_1, \ldots, i_n) \prod_{l=1}^{n} u(t - i_l) \tag{2}$$

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http://dx.doi.org/10.1016/j.apm.2013.10.041

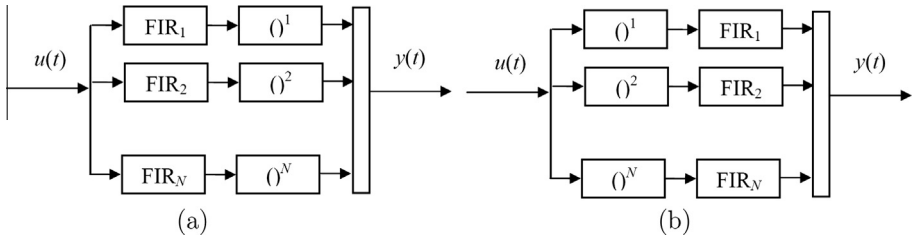

Fig. 1. Modified Generalized Memory Polynomial models (a) and Memory Polynomial (b).

is the n-dimensional convolution of the input $u \in \Re$ with the Volterra kernel h_n. This model, which is linear in the parameters, allows the convergence to a global optimum for any identification algorithm. However, the main drawback of this approach is the large number of parameters involved, giving rise to a high complexity model.

To overcome this problem, it is a common practice to appeal to some simplified DTVM models. For example, the use of Wiener or Hammerstein models are very frequent in many application fields ([6] and references therein). Although both Wiener and Hammerstein models are quite simple structures, they show a limitation as for the class of systems they can represent.

Recently, two appealing versions of the DTVM model have emerged in the context of compensation of nonlinear systems. Simplicity and generalization are their two outstanding properties. They are the Memory Polynomial (MP) [7,8] and the Modified Generalized Memory Polynomial (MGMP) [9]. These models allow the representation of a much more complex type of systems than Hammerstein and Wiener ones. In their typical form, these models adopt the structure depicted in Fig. 1.

The MP model is a generalization of a Hammerstein system [7], where multiple linear filters can be included after the static nonlinearity. Provided that the structure is formed by an Nth order polynomial and a finite impulse response (FIR) filter of order M, then the model output is given by

$$\hat{y}^{MP}(t) = \sum_{n=1}^{N}\left(\sum_{m=0}^{M-1}\alpha_{nm}u^{n}(t-m)\right), \tag{3}$$

where α_{nm} (with $n = 1,\ldots,N$ and $m = 1,\ldots,M$) stands for the compensator parameters. This structure has the advantage that the output signal is linear in the unknown parameters (i.e., the terms α_{nm}), which renders efficient parameter estimation which can be straightforwardly performed through least-squares methods.

An alternative is the MGMP model, it can be shown that in this case, the input–output relationship is given by

$$\hat{y}^{MGMP}(t) = \sum_{n=1}^{N}\left(\sum_{m=0}^{M-1}\alpha_{nm}u(t-m)\right)^{n}. \tag{4}$$

This model is the transpose of the block diagram presented for the MP model, with the power terms after the FIR filter in each parallel branch. Note that this model includes cross-terms among the samples $u(t-m)$. This is due to the fact that for a given n, the power terms in brackets at the left-hand side of Eq. (4) can be phrased as

$$\left(\sum_{m=0}^{M-1}\alpha_{nm}u(t-m)\right)^{n} \propto \sum_{m_1=0}^{M-1}\sum_{m_2=m_1}^{M-1}\cdots\sum_{m_n=m_{n-1}}^{M-1}\alpha_{nm_1}\cdots\alpha_{nm_n}u(t-m_1)\cdots u(t-m_n). \tag{5}$$

Both MP and MGMP models are special cases of finite Volterra models [3,7]. The model described by Eq. (5) is a generalization of a Wiener model different to the GMP derived in [7]. Note that the MGMP model output is not linear in the model parameters, this will lead, in general, to a complex identification algorithm.

A disadvantage of these models, based on FIR descriptions to represent dynamics behavior, is that they can demand too many parameters to describe those systems whose impulse response decays slowly. To cope with this fact, we propose the use of linear filters which incorporate prior knowledge about the process dynamics in order to reduce the number of parameters. Specifically, the FIR description is replaced by orthonormal Laguerre or Kautz filters.

The rest of the paper is organized as follows. In Section 2, the proposed model is defined, and the identification algorithms are presented. Section 3 states a discussion on the application of the proposed modeling approach for the identification of a distillation column. Finally, the paper ends with some conclusions in Section 4.

2. Volterra-type models

In this section, an alternative model is presented to represent multivariable nonlinear systems. The main underlying idea is to replace the tap delay used by the FIR filters depicted in Fig. 1 by an orthogonal basis [10,11].

Fig. 2 depicts the internal structure of the ith FIR filter shown in Fig. 1. On the other hand, Fig. 3 shows the internal structure when the proposed Laguerre bases replace the FIR filters.