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Optimal ordering policies for perishable multi-item under stock-dependent demand and two-level trade credit

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ABSTRACT

In this paper, a multi-item inventory model for perishable items is developed, where the demand rates of the items are stock dependent, two-level trade credit is adopted and the restriction of inventory capacity is also considered. The major objective is to determine the optimal cycle time and order quantities such that the total profit is maximized. The existence and uniqueness of the optimal cycle is discussed by Lagrange approach, and line search algorithms are developed to find the optimal solution of the model. Furthermore, numerical examples are given to illustrate the methods. The sensitivity of the solution to changes in the values of different parameters is also discussed.

1. Introduction

In the traditional inventory model, it was assumed that the retailer must pay his/her supplier as soon as he/she receives the ordered items. However, this is different with today's business transactions. Owing to the fierce market competition, credit sale becomes one of the main competitive means of the enterprises. General, the supplier allows a retailer to postpone paying money in a certain period (called a trade credit period). During the period, the retailer does not charge any interest and can earn interest by depositing the generated sales revenue into an interest bearing account. At the end of the period, the retailer settles the payment of the goods and has an interest charged for unsold goods. This is the so called trade credit or delay in payment. Taking the advantage of trade credit, the retailer reduces the cost and is motivated to order more quantities, which will increase the holding cost and the perishable cost. Therefore, the retailer has to balance between his/her revenue and expenditure. In recent studies on inventory management, several authors have examined the effect of trade credit policy on the optimal ordering policies. Goyal [1] first developed the economic order quantity (EOQ) inventory model under the condition of trade credit. Aggarwal and Jaggi [2] extended Goyal's model to deteriorating items. Jamal et al. [3] further generalized Goyal's model to allow for shortages. Chang et al. [4] developed an EOQ model for deteriorating items when the trade credit is linked to the order quantity. Hwang and Shinn [5] developed a model to determine optimal pricing and lot sizing for deteriorating items under condition of permissible delay in payments. Sarkar [6] discussed an EOQ model where demand and deterioration rate are both time dependent, and trade credit is included. There are several relevant papers related to trade credit such as Jamal et al. [7], Sana and Chaudhuri [8], Chung and Liao [9], Balkhi [10], Liao et al. [11], Shah et al. [12] and their references. All the above inventory models implicitly assumed that the supplier would offer the retailer a delay period but the retailer would not offer any delay period to his/her customer. In most business transactions,

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this assumption is unrealistic. Usually the supplier offers a credit period to the retailer and the retailer, in turn, passes on this credit period to his/her customers [13]. That is so called two-level trade credit. Huang [14] presented an inventory model assuming that the retailer also permits a credit period to its customer which is shorter than the credit period offered by the supplier. Huang [15] extended Huang's model to investigate the inventory policy under two-level trade credit and limited storage space, in which there two warehouses, one is own and another is rented. Kreng and Tan [16] developed an inventory model under two-level credit trade policy when the credit period depends on the order quantity. Ho [17] considered a supply chain system under two-level credit trade when the demand depends on price and credit period offered by supplier. Chen and Kang [18] established a supply chain system with the assumption that demand is a negative exponential function of price and two-level credit trade is adopted. Liao [19] developed an EOQ model with non-instantaneous receipt and exponentially deteriorating items under two-level trade credit.

An interesting phenomenon is observed in the supermarket that display of the consumer goods in large quantities attracts more customers and generates higher demand. Many researchers have given consider able attention to the situation where the demand rate is dependent on the level of the on-hand inventory. Baker and Urban [20] established an economic order quantity model by assuming that the demand rate was a function of the instantaneous stock level. Chang et al. [21] developed inventory model with a deteriorating item and stock-dependent demand rate. Sajadieh et al. [22] reported a supply chain system with stock-dependent demand rate. Jolai et al. [23] established a model under inflation for deteriorating items with stock-dependent rate. Hsieh et al. [24] considered a model with stock-dependent demand rate under the assumption that backlog rate is a function of the waiting time of customer. Bhattacharya [25] developed a two-item model for deteriorating items, the demand of one item depended on the other's stock level. Kar et al. [26] proposed a model that there are fresh and deteriorating items sold from the primary and the secondary shop respectively. The demand of fresh items depends on selling price and stock level. Recently, Min et al. [27] established a single item inventory model with deteriorating item, stock dependent demand rate and two level trade credits. All above models are developed for a single item. However, in real life, many companies, enterprises or retailers deal with several items and stock them in their showroom/warehouse for sale. There is a restriction either on maximum capital investment in stock at any time, or the maximum warehouse space available for storage. Padmanabhan and Vrat [28] developed multi-item multi-objective inventory model of deteriorating items with stock-dependent demand. Ben-Daya and Raouf [29] discussed a multi-item inventory model with stochastic demand subject to the restrictions on available space and budget. Saha et al. [30] developed a multi-item inventory model with the breakability rate and the demand rate both stock-dependent. Tsao [31] considered multi-echelon multi-item channels subject to supplier's credit period and retailer's promotional effort. Tsao and Sheen [32] developed a multi-item supply chain with credit periods and weight freight cost discounts. Thangam and Uthayakumar [33] developed an EPQ-based model for perishable items with two-level trade credit and demand rate both selling price and credit period dependent. Thangam [34] considered a supply chain for perishable items under advance payment scheme and two-level trade credit. Su [35] established an integrated inventory system with defective items and allowable shortage under trade credit. The aforesaid multi-item inventory models were developed with either stock dependent or trade credit.

In this paper, we discussed the optimal order policy for retailer with perishable multi-item and stock-dependent demand rate under two-level trade credit and restriction on available space or budget. This is basically an extension of the single item inventory model by Min et al. [27] to deal with multiple items and restriction on inventory capacity. In more details, we are taking into account the following factors: (1) a supplier sells multiple items to a retailer; (2) the selling items are perishable such as meats, fruits, green vegetables, foodstuffs, etc.; (3) the demand rate of each item is dependent on its instantaneous stock level; (4) the supplier provides the retailer a trade credit period (M) for payments and the retailer offers the partial trade credit period (N) to his/her customers, and $N \leqslant M$; (5) the retailer has a restriction on available space or budget. Under these conditions, we model the retailer inventory system as a constraint optimization problem. The aim is to determine the optimal ordering polices to maximize the average system profit. The existence and uniqueness of optimal strategy are discussed by the Lagrange method and line search algorithms are presented to find the optimal cycle length and lot size. Finally, numerical examples are given to illustrate the theoretical results and the methods.

2. Notations and assumptions

2.1. Notations

i	the index of products, $i = 1, 2, \ldots, n$
K	ordering cost of one order
c_i	unit purchasing price of product i
h_i	unit stock holding cost per year of product i
p_i	unit selling price of product i
w_i	unit capacity of product i
W	the total storage capacity of inventory

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