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Impulsive perturbations in a predator–prey model with dormancy of predators

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ABSTRACT

In this paper, a predator–prey model consisting of active and dormant states of predators with impulsive control strategy is established. Using Floquet theories, the small amplitude perturbation technique and the piecewise Lyapunov function method, the conditions of local and global asymptotical orbital stability of the prey-eradication periodic solution are obtained. The boundness and permanence of the impulsive system are proved by the comparison principle. Through numerical simulations, the effects of the impulsive perturbation on the inherent oscillation are investigated, which implies that the impulsive perturbation can lead to period-doubling bifurcation, chaos, and period-halving bifurcation. Moreover, the effects of the impulsive perturbation and hatching rate on the chaos of the system are comparatively studied by numerical simulation. These obtained results can be useful for ecosystem management and for explaining complex phenomena of ecosystems.

1. Introduction

Recently, the effects of impulsive perturbations on population systems have been widely studied and discussed by a number of researchers [1–9]. Ecologists use more impulsive phenomena to develop and exploit biological resources. In agriculture, there are two methods for pest control. One is using insecticides to kill pests at a fixed time, and the other is to regularly put natural enemies to eliminate pests.

Controlling algae blooms is a very important issue for ecosystem management. There are different approaches to get rid of algae. The direct means include physical and chemical methods. Chemical control relies mainly on the use of synthetic algicide to suppress algae. Algicide are useful because they quickly kill a significant portion of an algae population and they sometimes provide the only feasible method for preventing economic loss. However, algicide pollution is also recognized as a major health hazard to human beings and to natural enemies. Biological control is the reduction in algae by releasing other living organisms, which eat algae such as *Daphnia* and some fish. An experiment [10] shows that the amplitude of prey-predator cycles of *Daphnia* and its algae prey in microcosms increases when a portion of ephippia-producing females is replaced by asexually-reproducing gravid females. This suggests that the dormancy of predators can influence the population dynamics of *Daphnia* and its algae prey at high nutrient levels. Base on this idea, Kuwamura [11] proposed a minimum mathematical model of predator–prey system with the dormancy of predators to explain the paradox of enrichment in ecosystems. In this paper, we will take account of constant impulsive perturbations of an active predator into the model in [11]. The model can be described by the following impulsive differential equations:

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$$\begin{cases} \dot{p} = r\left[1 - \frac{p(t)}{\widetilde{K}}\right]p - f(p)z_1, & t \neq n\tau, \\ \dot{z_1} = k_1\mu(p)f(p)z_1 + \tilde{\alpha}z_2 - \tilde{d}_1 z_1, & t \neq n\tau, \\ \dot{z_2} = k_2(1-\mu(p))f(p)z_1 - \tilde{\alpha}z_2 - \tilde{d}_2 z_2, & t \neq n\tau, \\ p(n\tau^+) = p(n\tau), \\ z_1(n\tau^+) = z_1(n\tau) + \tilde{\rho}, \\ z_2(n\tau^+) = z_2(n\tau), \end{cases} \tag{1.1}$$

where p and z_1 denote the prey and predator densities, respectively; z_2 denotes the density of predators with dormant state (resting eggs); r and $\widetilde{K}$ are the intrinsic growth rate and the carrying capacity of prey, respectively; $f(p) = \frac{bp}{c+p}$ is the Holling type II functional response (b and c denote the maximum foraging and the half saturation constant, respectively); k_1 and k_2 are increasing rates of predator in active and dormant states, respectively; $\mu(p) = \frac{\tanh(\frac{p-\tilde{\eta}}{\tilde{\sigma}})+1}{2}$ and $1-\mu(p)$ are the switching function of subitaneous (active state of predator) and resting eggs (dormant state of predator), respectively ($\tilde{\eta}$ is a certain switch level, $\tilde{\sigma}$ is the sharpness of the switching effect); $\tilde{d}_1$ and $\tilde{d}_2$ denote the mortality rates of the active and dormant predator, respectively; $\tilde{\alpha}$ denotes the hatching rate (see [11] for more details about the model). In addition, τ is the period of the impulsive effect, $n \in \mathbb{N}$, $\mathbb{N}$ is the set of all non-negative integers, $\tilde{\rho} > 0$ is the release amount of predator at $t = n\tau$.

For system (1.1), with a nondimensionalized change of similar variables transform (see [12,13], Introduction), denoting $s = rt$ (and then still denoting s by t), $x_1(t) = \frac{p(t/r)}{c}$, $x_2(t) = \frac{2z_1(t/r)}{ck_1}$, $x_3(t) = \frac{2z_2(t/r)}{ck_2}$, $\eta = \frac{\tilde{\eta}}{c}$, and using the following notations: $K = \widetilde{K}c^{-1}$, $m = \frac{bk_1}{2r}$, $\theta = \frac{k_2}{k_1}$, $\alpha = \frac{\tilde{\alpha}}{r}$, $d_1 = \frac{\tilde{d}_1}{r}$, $d_2 = \frac{\tilde{d}_2}{r}$, $\sigma = \frac{c}{\tilde{\sigma}}$, $T = r\tau$, $\rho = \frac{2\tilde{\rho}}{ck_1}$, then we obtain the simplified dimensionless system:

$$\begin{cases} \dot{x_1} = \left(1 - \frac{x_1}{K}\right)x_1 - \frac{mx_1x_2}{1+x_1}, & t \neq nT, \\ \dot{x_2} = [1 + \tanh(\sigma(x_1 - \eta))]\frac{mx_1x_2}{1+x_1} + \theta\alpha x_3 - d_1x_2, & t \neq nT, \\ \dot{x_3} = [1 - \tanh(\sigma(x_1 - \eta))]\frac{mx_1x_2}{1+x_1} - (\alpha + d_2)x_3, & t \neq nT, \\ x_1(nT^+) = x_1(nT), \\ x_2(nT^+) = x_2(nT) + \rho, \\ x_3(nT^+) = x_3(nT). \end{cases} \tag{1.2}$$

The solution of system (1.2) is a piecewise continuous function $X(t) : \mathbb{R}_+ \longrightarrow \mathbb{R}^3_+$ which is continuous on $(nT, (n+1)T], n \in \mathbb{N}$ and $X(nT^+) = \lim_{t \to nT^+} X(t)$ exists, where $X(t) = (x_1, x_2, x_3), \mathbb{R}_+ = [0, \infty), \mathbb{R}^3_+ = \{X(t) \in \mathbb{R}^3 : X(t) \geqslant 0\}$. Obviously the smoothness properties guarantee the global existence and uniqueness of solutions of system (1.2). (See [14,15] for details on fundamental properties of impulsive systems.)

In this paper, we mainly study the effects of impulsive perturbations on a predator–prey model with dormancy of predators.

The rest of the paper is organized as follows. In Section 2, for the convenience of investigation, we give some notations and definitions. In Section 3, we prove that the prey-eradication periodic solution is locally and globally asymptotically orbitally stable, and system (1.2) is uniformly ultimately bounded and permanent. In Section 4, using numerical simulations, we study the effects on the inherent oscillation caused by the impulsive perturbations as well as the dynamical effects of the impulsive perturbation and hatching rate on the chaos of the system. Biological implications of our results and further discussions are given in Section 5.

2. Notations and definitions

Next we introduce some common definitions about impulsive differential equations. Let,

$$V_0 = \left\{ V : \mathbb{R}_+ \times \mathbb{R}^3_+ \longrightarrow \mathbb{R}_+, \text{ continuous on } (nT, (n+1)T] \times \mathbb{R}^3_+, \text{ and } \lim_{(t,Y) \to (nT^+, X)} V(t, Y) = V(nT^+, X) \text{ exists} \right\}$$

be a piecewise Lyapunov function, and its upper right derivative (see [6]) is as follows.

Definition 2.1. Let $V \in V_0$. Then for $(t, X) \in (nT, (n+1)T] \times \mathbb{R}^3_+$, the upper right derivative of $V(t, X)$ with respect to the impulsive differential system (1.2) is defined as:

$$D^+V(t, X) = \lim_{h \to 0+} \sup \frac{1}{h}[V(t+h, X + hf(t, X)) - V(t, X)].$$

Definition 2.2. System (1.2) is regarded as permanent if there exist constants $T_0, M, m > 0$, satisfying $m \leqslant x_i(t) \leqslant M, i = 1, 2, 3$ when $t > T_0$, where $X(t)$ is any solution of system (1.2) with the initial values $x_i(0^+) > 0, i = 1, 2, 3$.

In studying the boundness and permanence, we will use the following comparison principle from Lemma 2.2 in [6].