Applied Mathematical Modelling 38 (2014) 2601–2612

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

A hybrid of adjustable trust-region and nonmonotone algorithms for unconstrained optimization

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ARTICLE INFO

Article history:
Received 21 August 2012
Received in revised form 20 May 2013
Accepted 11 October 2013
Available online 16 November 2013

Keywords:
Unconstrained optimization
Trust-region framework
Nonmonotone strategy
Adjustable radius
Global convergence

ABSTRACT

This study devotes to incorporating a nonmonotone strategy with an automatically adjusted trust-region radius to propose a more efficient hybrid of trust-region approaches for unconstrained optimization. The primary objective of the paper is to introduce a more relaxed trust-region approach based on a novel extension in trust-region ratio and radius. The next aim is to employ stronger nonmonotone strategies, i.e. bigger trust-region ratios, far from the optimizer and weaker nonmonotone strategies, i.e. smaller trust-region ratios, close to the optimizer. The global convergence to first-order stationary points as well as the local superlinear and quadratic convergence rates are also proved under some reasonable conditions. Some preliminary numerical results and comparisons are also reported.

1. Introduction

In this study, we consider the following unconstrained optimization problem

$$\text{Minimize } f(x), \quad \text{subject to } x \in \mathbf{R}^n, \tag{1}$$

where $f : \mathbf{R}^n \to \mathbf{R}$ is a real-valued twice continuously differentiable function. Two principle schemes have been developed for solving this problem, namely, line search and trust-region methods [1,2]. The idea of line search methods is to find a steplength along a specific direction, but trust-region methods are a prominent class of methods for unconstrained optimization problems defining an area around the current step x_k in which the quadratic model has a good agreement with the objective function. In these methods, in each iterate, a trial step d_k is obtained by solving the following quadratic subproblem

$$\text{Minimize } m_k(d) = f_k + g_k^T d + \frac{1}{2} d^T B_k d, \quad \text{subject to } d \in \mathbf{R}^n \quad \text{and} \quad \|d\| \leqslant \delta_k, \tag{2}$$

in which $\|.\|$ is the Euclidean norm, $f_k = f(x_k), g_k = \nabla f(x_k), B_k$ is $H_k = \nabla^2 f(x_k)$ or its symmetric approximation, and δ_k is the trust-region radius. A crucial point in trust-region methods is a strategy of choosing the trust-region radius δ_k, at every iterate. In the standard trust-region method, based on agreement between the model and the objective function, the radius of trust-region is updated by paying attention to the following ratio

$$r_k = \frac{f(x_k) - f(x_k + d_k)}{m_k(0) - m_k(d_k)}. \tag{3}$$

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http://dx.doi.org/10.1016/j.apm.2013.10.062

The numerator and the denominator of (3) have been called the actual reduction and the predicted reduction, respectively. It can be concluded that there will be a good agreement between the model and the objective function over current region of trust whenever r_k be close to 1. In this case, it is safe to increase the trust-region radius in the next iterate. Otherwise, the trust-region radius must be shrunk.

It is well-known that the standard trust-region method is very sensitive on the initial trust-region radius, see for example [3–5]. In other word, we know that the standard trust-region radius δ_k is independent from g_k and B_k, so we do not know the radius δ_k is suitable to the whole of implementation. This situation possibly increases the number of solving subproblems in the inner steps of the method and so decreases the efficiency of the method. It is obvious that if we decrease the number of ineffective iterates, we can decline the number of solving subproblems in each step. In [4], Sartenaer proposed an approach to determine the initial radius monitoring agreement between the model and the objective function along the steepest descent direction computed at the starting point. The first adjustable strategy to determine the trust-region radius, for decreasing the number of solving subproblems, was proposed by Zhang et al. in [6]. This strategy used the information of gradient and Hessian in current iterate to construct the trust-region radius δ_k without requiring any initial trust-region radius. Inspired by Zhang strategy, Shi and Guo in [5] proposed a automatically adjustable radius for trust-region methods. They proved that the new method preserves the global, the superlinear and the quadratic convergence properties of the standard method. The numerical experiments have been showed that this method is more efficient than Zhang's method and standard trust-region method. We describe these trust-region radiuses in the next section.

On the other hand, Grippo, Lampariello and Lucidi in [7,8] provided a nonmonotone strategy to line search methods for unconstrained optimization problems. In their nonmonotone line search, steplenght θ_k is accepted if it satisfies the following Armijo-type condition

$$f(x_k + \theta_k d_k) \leqslant f_{l(k)} + \sigma\theta_k \nabla f(x_k)^T d_k, \tag{4}$$

where $\sigma \in (0,1)$, and

$$f_{l(k)} = \max_{0 \leqslant j \leqslant m(k)} \{f_{k-j}\}, \quad k = 0,1,2,\ldots, \tag{5}$$

in which $m(0) = 0$ and $m(k) = min\{m(k-1)+1, N\}$ with an integer constant $N \geqslant 0$. Theoretical analysis and numerical experiments have been indicated the efficiency and robustness of this strategy to improve both the possibility of finding the global optimum and the rate of convergence of algorithms [9]. Motivated by these outstanding results, many researchers have interested to work on combination the nonmonotone strategy with the trust-region methods [10,9,11,12]. Hence various versions of the nonmonotone trust-region methods have been proposed so far.

The nonmonotone techniques have changed the ratio (3) comparing the actual reduction with the predicted reduction. One of the most common nonmonotone ratios is defined as follows

$$\tilde{r}_k = \frac{f_{l(k)} - f(x_k + d_k)}{m_k(0) - m_k(d_k)}. \tag{6}$$

It has been proved that the nonmonotone trust-region methods inherit the strong convergence properties of the standard trust-region method. Their numerical experiments have been showed that the nonmonotone trust-region methods are more efficient than the monotone versions, especially in a presence of the narrow curved valley. Although, the nonmonotone strategy of Grippo, Lampariello and Lucidi performs very well in many cases, it contains some drawbacks, including line search or trust-region. Two important instances of these drawbacks can describe as follows

- A good function value generated in any iterate is essentially discarded due to "max" term in (5).
- In some cases, the numerical performances seriously depend on the choice of parameter N.

Zhang and Hager in [13] proposed a new nonmonotone strategy for the line search methods based on a weighted average of former successive iterates. This strategy is generally efficient and promising when encounter with the unconstrained optimization and can overcome mentioned drawbacks. The method relaxes the Armijo condition (4) by substituting $f_{l(k)}$ with a weighted average of previous successive iterates C_k which is defined as follows

$$C_k = \begin{cases} f(x_k), & \text{if } k = 0; \\ (\eta_{k-1} Q_{k-1} C_{k-1} + f(x_k))/Q_k, & \text{if } k \geqslant 1, \end{cases}$$
$$Q_k = \begin{cases} 1, & \text{if } k = 0; \\ \eta_{k-1} Q_{k-1} + 1, & \text{if } k \geqslant 1, \end{cases} \tag{7}$$

where $0 \leqslant \eta_{min} \leqslant \eta_{k-1} \leqslant \eta_{max} \leqslant 1$. Recently, Mo, Liu and Yan in [14] take advantages of the nonmonotone strategy of Zhang and Hager in trust-region framework to propose a new nonmonotone trust-region method. In their proposal, the ratio (6) changed as

$$\hat{r}_k = \frac{C_k - f(x_k + d_k)}{m_k(0) - m_k(d_k)}. \tag{8}$$