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Exact algorithms for multi-criteria multi-modal shortest path with transfer delaying and arriving time-window in urban transit network

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ABSTRACT

This paper investigates the solution algorithms for the multi-criteria *multi-modal shortest path problem* (M-SPP), which belongs to the set of problems known as NP-hard, in *urban transit network* (UTN). The related M-SPP is one of the important and practical problems in several fields such as urban transportation system and freight transportation. The UTN is composed of multiple modes (e.g., automobile, bus, subway, light rail, pedestrian and so on). To get their destination, the passengers can alternate between different modes. As a special demand, the time-window is usually associated with the M-SPP. Because of the service time-limit of modes, the available modes at a stop are varied with the time. So the optimal M-SPP with arriving time-window cannot be simply obtained by finding the optimal M-SPP firstly and then reversely deducing the leaving time-window of the origin according to the arriving time-window of destination. In this paper, the M-SPP with arriving time-window is firstly proposed. To solve the *multi-criteria* M-SPPs (MM-SPP) with transfer delaying, an improved exact *label correcting algorithm* (LCA) is designed and, to solve the proposed MM-SPPs with both of transfer delaying and arriving time-window, an exact reverse LCA is designed. Finally, some computing examples are given to test the effectiveness of the designed algorithms.

1. Introduction

The main purpose of this paper is to investigate the exact algorithms for solving the *multi-criteria* M-SPP (MM-SPP) to minimize the total travel time, in which the delaying time at transfer stop and the arriving time-window of destination are considered, and total travel cost in *urban transit network* (UTN). It is well known that the *shortest path problem* (SPP) is one of the oldest and widely applied problems in network optimization [1,2]. Follows from the practical point of view, the SPP and its variations have been extensively investigated in different fields. One of the most interesting application domains is the transportation in which the problems in various forms such as the classic, constrained, static, dynamic, stochastic and multi-criteria can be recognized [3]. The M-SPP is an important extending of SPP to UTN and freight transportation system. Another more practical extension of M-SPP in UTN is the M-SPP with arriving time-window of destination.

Multi-modal transportation is the combination of two or more means of transport to move passengers or goods from a predetermined origin to a predetermined destination [4,5]. The passengers can use either public vehicles (e.g., bus, taxi, metro, subway, light rail, and ship) or private vehicles (e.g., car, motorbike, bicycle, and walking). Multi-modal routes are

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becoming a real necessity in our society in order to guarantee a high level of mobility both inside cities and at the regional level [6]. In other words, M-SPP is concerned with finding a path from a predetermined origin (O) to a predetermined destination (D) and the vehicles transfer schedule on this path in a given multi-modal network such that the total cost associated with the multi-modal path is the optimum (minimum). The complexity of finding M-SPP is obviously much higher than mono-modal one (i.e., the classical SPP). Furthermore, the service time-limit of modes isn't considered in the existing algorithms for the M-SPP. In this paper, the service time-limit of modes is taken into consideration and the related problems are those to minimize both of the total travel time, which includes the delaying time and in-vehicles time, and the total travel cost. It is known to us that the multi-criteria SPP is that belongs to the set of problems known as NP-hard [7], so the related MM-SPP is also that of NP-hard.

Among the most practically M-SPPs, the time-window is proposed in some special situations. For example, the passenger who wants to take the railway train or airplane usually pays more attentions to the arriving time or, more practically, the arriving time-window of the destination (i.e., a railway train station or an airport). For the M-SPP with arriving time-window in UTN, the decision not only includes that to find a feasible M-SPP with minimal travel time and travel cost, but also to determine the feasible leaving time-window of the origin stop. Because of the service time-limit of modes, i.e., the first vehicle hour and final vehicle hour, of some modes in UTN, each stop-mode pair is usually associated with a time-window to indicate its available service time-limit. Meanwhile, according to the arriving time-window of destination, each stop-mode pair is simultaneously associated with another feasible time-window to take the related mode at this stop. So the algorithm for M-SPP with arriving time-window cannot be simply obtained by finding the M-SPP with minimum travel time and minimum travel cost and then reversely deducing the leaving time-window of the origin according to the arriving time-window of destination. Briefly speaking, the M-SPP with arriving time-window is to find an optimal multi-modal path and the available leaving time-window of the origin according to the arriving time-window of destination. The M-SPP with arriving time-window is a special constrained shortest path problem. The general constrained shortest path problem is a problem that belongs to the set of problems known as NP-hard [7]. This paper will focus on investigating an exact algorithm for M-SPP with both of transfer delaying and arriving time-window.

Generally, there are two principal classes of algorithms for solving multi-objective SPPs with linear objective functions: Labeling based algorithms and ranking path based algorithms. The former may be divided into two main families: The *label setting algorithms* (LSA) (e.g., [8,9]) in which one label is set permanently at each iteration, and the *label correcting algorithms* (LCA) (e.g., [10,11]) in which all the labels become permanent only in the last iteration. In this paper, the label correcting approach is adopted to design the related algorithms. Generally, M-SPPs are categorized into the static and the dynamic. In this paper, the related problem is assumed to be the static, i.e., the weights of arcs are determinate.

In this paper, by using the label correcting approach and designating a $(p+3)$-tuple labels set to each mode stop, an improved exact algorithm is designed for solving MM-SPPs with transfer delaying to minimize the total travel time and overall travel cost. Additionally, we investigate the MM-SPP with both of the arriving time-window of destination and transfer delaying and, by using the reverse label correcting approach, an exact algorithm is designed for solving the MM-SPP with both of the arriving time-window of destination and transfer delaying time at transfer stop.

This paper is organized as follows: Firstly, the literature of M-SPP is reviewed in Section 2. The notations associated with M-SPP and multi-criteria optimization are introduced in Sections 3.1 and 3.2, respectively. The related multi-criteria models are formulated in Sections 3.3 and the label correcting solution algorithms are designed in Section 4. Finally, the proposed algorithms are tested in Section 5.

2. Literature review

There have been many solution algorithms for M-SPP. Most of them are those to find solutions for static M-SPP [12–15]. The waiting time of transfer was usually approximated according to headway in most of these algorithms. Recent researches pay more attentions on dynamic solutions. Generally speaking, the M-SPPs can be hierarchically categorized, where the upper level includes static and dynamic and the lower level includes single objective, multi-criteria, constrained, stochastic and so on. The problems in real word are the combination of these different categories.

Based on the classical shortest path problem, Modesti and Sciomachen [16] presented an approach for finding multi-objective shortest paths in multi-modal networks with objectives of minimizing the overall cost, time and users' discommodity associated with the required paths. They used an ad hoc utility function to assign the weights to arcs according to their cost and time while simultaneously considered the preferences of the users related to all of the possible transportation modalities.

Lozano and Storchi [17] proposed a multi-modal shortest path algorithm to find the shortest viable path, in which paths with illogical sequences of used modes were eliminated. They used label correcting techniques and an ad hoc modification of the chronological algorithm which was proposed by Pallottino and Scutella [3] to solve the problem. They extended their algorithm to calculate the viable hyperpath [18].

Bielli et al. [6] proposed a framework to address both algorithmic approaches proposed for solving the multi-modal shortest path problem and a transportation network modeling using *geographic information systems* (GIS). They developed a modified version of *k*-shortest path algorithm to define an efficient solution for multi-modal shortest path with time constraints. A new graph structure, namely transfer graph, was introduced by Ayed et al. [19] to model the multi-modal