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Hybrid flexible flowshop problems: Models and solution methods

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ABSTRACT

This paper considers the problem of hybrid flowshop scheduling. First, we review the shortcoming of the available model in the literature. Then, four different mathematical models are developed in form of mixed integer linear programming models. A complete experiment is conducted to compare the models for performance based on the size and computational complexities. Besides the models, the paper proposes a novel hybrid particle swarm optimization algorithm equipped with an acceptance criterion and a local search heuristic. The features provide a fine balance of diversification and intensification capabilities for the algorithm. Using Taguchi method, the algorithm is fine tuned. Then, two numerical experiments are performed to evaluate the performance of the proposed algorithm with three particle swarm optimization algorithms available in the scheduling literature and one well-known iterated local search algorithm in the hybrid flowshop literature. All the results show the high performance of the proposed algorithm.

1. Introduction

Among the different complex combinatorial optimization problems, shop scheduling problems are uttermost active field of research. Their applications could be found in wide variety of situations, from information services to manufacturing systems [1]. Among different shop scheduling systems, a flowshop problem is a multi-stage manufacturing process in which each stage has a single processor. The flowshop occurs when manufacturing products (or performing jobs) with the same generic recipe.

In real world cases, we rarely encounter a shop with a single processor at each stage. Commonly, processors are duplicated in parallel at stages. The purpose is to balance the capacity of stages, increase the overall shop floor capacity, reduce, if not eliminate, the impact of bottleneck stages and so on [2], Behnamian and Fatemi Ghomi [3]. The shop with multi processors at its stages (or at least one stage with more than one processor) is called a hybrid flowshop. A classical assumption in flowshop related problems is that all jobs need to visit all stages. Yet in many applications, each job might skip some stages. Considering stage skipping results in a shop called hybrid flexible flowshop (HFF). One can find applications of HFF in food processing industry, ceramic tile manufacturing, the processing of wood and the manufacture of furniture.

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Commonly, mathematical programming models, in form of mixed integer linear programming (MILP), have been proposed and evaluated for finding optimal solutions for scheduling problems Naderi and Ruiz [2]. Apart from MILP model development, many papers review and evaluate the available MILP models for regular flowshops [4–6], job shops [4], open shops [7]. However, as we explain next, there is very limited available literature on this important topic of MILP development for HFF or relevant problems, let alone comparative studies. Even, this sporadic literature suffers from several critical shortcomings that this paper aims to address. The purposes of this paper are three-fold. First, the shortcomings of available MILP models for HFF are described in summary fashion. Second, four different MILP models are developed. Third, all four models are compared for performance.

Moreover, to solve larger sizes of the problems, we propose a novel particle swarm optimization (PSO). To evaluate the proposed algorithm, we compare it with four available algorithms. They are three available versions of particle swarm optimization (combinatorial, discrete, and hybrid PSOs) and one iterated local search algorithm.

The rest of the paper is organized as follows. Section 2 formally defines the problem and its existing literature. Section 3 develops four different MILP models for the problem under consideration. Section 4 compares the models for performance.

2. Problem definition and available literature

A hybrid flexible flowshop problem could be described as follows. There is a set N of n jobs and a set M of m stages where at stage i ($i = \{1, 2, \ldots, m\}$) there are a set of m_i identical machines. Every job j ($j = \{1, 2, \ldots, n\}$) is required to follow the exact same processing sequence across all stages, starting from stage 1, then stage 2 until stage m. Each job might skip some stages. Therefore, each stage i processes a set E_i of e_i jobs where $E_i \subseteq N$. Each job is processed by exactly one machine l ($l = \{1, 2, \ldots, m_i\}$) among machine available at each stage where $p_{j,i}$ denotes the processing time of job j at stage i. Since machines inside each stage are identical, the processing time is only job-stage dependent.

Each machine can process no more than one job simultaneously while each job can be processed by no more than one machine at a time. The setup and transportation are negligible or included into processing times. There is no machine failure; hence, machines are continuously available for processing. Finally, All jobs are available at time 0 and the process of a job on a machine can be never interrupted; therefore, once the process starts, it continues until it finishes. The objective is to both assign jobs to one machine at each stage and then sequence jobs on machines so as to minimize the maximum completion time of jobs, called makespan.

The problem of HFF with separated setup time is formulated by Kurz and Askin [8]. The model does not consider the assignment since its variables only find the sequence. In this model, the constraint set (5) specifies that completion time of two consecutive jobs at each stage. Therefore, it is not extendable for the case of hybrid flow shop with non-identical machines. The problem of HFF with transportation time is formulated by Naderi et al. [9]. This model is also based on the model of Kurz and Askin [8] and suffers from the same flaw.

Another attempt to formulate HFF is done by Paternina-Arboleda et al. [10]. In this model, although the assignment of jobs to machines at each stage is determined by constraint set (4), yet it never uses it in constraint sets (6) and (7) to prevent from making relation between completion times of two jobs if they are assigned to two different machines at one stage. Therefore, it suffers the same flaw as the other models do. Behnamian and Zandieh (2011) present another model which suffers from several mistakes.

Kis and Pesch [11] propose a MILP model for hybrid flowshops with no flexibility. Another model with the same concept is proposed for HFF with some additional assumptions by Ruiz et al. [12]. Unfortunately, although this model works correctly, it is not effective due to its large complexity size. This issue will be discussed later in next two sections. As reviewed, one can see the available models suffer from serious flaws and shortcomings. This lack makes a research on MILP formulation of HFF more and more interesting.

3. Mixed integer linear programming models

This paper proposes three different novel models for HFF. The application of integer programming models in solving scheduling problems starts with the pioneer model of Wagner [13]. Yet, regarding the limitation of computer capacity and the lack of specified software, the progress of research on this field is not as active as the other solution approaches. Due to recent advances obtained in computer capacity and advent of efficient specialized software, the MILP model development is each time becoming more and more interesting. Even if we accept this idea that mathematical models cannot be efficient solution algorithms, they are the first natural way to approach scheduling problems by Pan [4]. They can explicitly describe all the characteristics of a scheduling problem. Furthermore, mathematical models are used in many solution methods such as branch and bound, dynamic programming and branch and price. More efficient MILP models would result in more effective solution methods.

The models presented here are on three different bases that are separately discussed in the following. Apart from the three proposed model, the only available model is also adapted to problem under study. The parameters and indexes used in these models are: