Applied Mathematical Modelling 38 (2014) 5781–5795

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Macroscopic modeling and simulations of room evacuation

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ARTICLE INFO

Article history:
Received 4 August 2013
Received in revised form 23 January 2014
Accepted 18 March 2014
Available online 14 May 2014

Keywords:
Macroscopic models
Crowd dynamics
Evacuation
Braess paradox

ABSTRACT

We analyze numerically two macroscopic models of crowd dynamics: the classical Hughes model and the second order model being an extension to pedestrian motion of the Payne–Whitham vehicular traffic model. The desired direction of motion is determined by solving an eikonal equation with density dependent running cost, which results in minimization of the travel time and avoidance of congested areas. We apply a mixed finite volume-finite element method to solve the problems and present error analysis for the eikonal solver, gradient computation and the second order model yielding a first order convergence. We show that Hughes' model is incapable of reproducing complex crowd dynamics such as stop-and-go waves and clogging at bottlenecks. Finally, using the second order model, we study numerically the evacuation of pedestrians from a room through a narrow exit.

1. Introduction

Crowd dynamics has recently attracted the interests of a rapidly increasing number of scientists. Analytical and numerical analysis are effective tools to investigate, predict and simulate complex behavior of pedestrians, and numerous engineering applications welcome the support of mathematical modeling. Growing population densities combined with easier transport lead to greater accumulation of people and increase risk of life threatening situations. Transport systems, sports events, holy sites or fire escapes are just few examples where uncontrolled behavior of a crowd may end up in serious injuries and fatalities. In this field, pedestrian traffic management is aimed at designing walking facilities which follow optimal requirements regarding flow efficiency, pedestrians comfort and, above all, security and safety.

From a mathematical point of view, a description of human crowds is strongly non standard due to the intelligence and decision making abilities of pedestrians. Their behavior depends on the physical form of individuals and on the purpose and conditions of their motion. In particular, pedestrians walk with the most comfortable speed [1], tend to maintain a preferential direction towards their destination and avoid congested areas. On the contrary, in life threatening circumstances, nervousness make them move faster [2], push others and follow the crowd instead of looking for the optimal route [3]. As a consequence, critical crowd conditions appear such as "freezing by heating" and "faster is slower" phenomena [4,5], stop-and-go waves, transition to irregular flow [6] and arching and clogging at bottlenecks [2].

In order to describe this complex crowd dynamics, numerous mathematical models have been introduced, belonging to two fundamentally distinct approaches: microscopic and macroscopic. In the microscopic framework pedestrians are treated as individual entities whose trajectories are determined by physical and social laws. Examples of microscopic models are the social force model [7], cellular automata models [8,9], AI-based models [10]. Macroscopic description treats the crowd as a

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http://dx.doi.org/10.1016/j.apm.2014.03.027
0307-904X/

continuum medium characterized by averaged quantities such as density and mean velocity. The first modeling attempt is due to Hughes [11] who defined the crowd as a "thinking fluid" and described the time evolution of its density using a scalar conservation law. Current macroscopic models use gas dynamics equations [12,13], gradient flow methods [14], non linear conservation laws with non classical shocks [15] and time evolving measures [16]. At an intermediate level, kinetic models derive evolution equations for the probability distribution functions of macroscopic variables directly from microscopic interaction laws between individuals, see for example, [17,18] and references therein. Also, recently introduced approaches include micro–macro coupling of time-evolving measures [19] and mean-field games [20]. These models are good candidates to capture the effects of individual behavior on the whole system.

In this paper we shall analyze and compare two macroscopic models describing the time evolution of the density of pedestrians. The first one, introduced by Hughes [11], consists of a mass conservation equation supplemented with a phenomenological relation between the speed and the density of pedestrians. The second one involves mass and momentum balance equations so is of second order type. It was proposed by Payne [21] and Whitham [22] to describe vehicular traffic and adopted to describe pedestrian motion by Jiang et al. [13]. It consists of the two-dimensional Euler equations with a relaxation source term. In both models, the pedestrians' optimal path is computed using the eikonal equation as was proposed by Hughes [11].

In order to simulate realistic behavior we consider two dimensional, continuous walking domains with impenetrable walls and exits as pedestrians' destination. To our knowledge the only available results using Hughes' model concern simulations of flow of pedestrians on a large platform with an obstacle in its interior [23,24]. In the case of the second order model Jiang et al. [13] considered the same setting and showed numerically the formation of stop-and-go waves. However, none of the above works analyzed complex crowd dynamics. Behavior at bottlenecks and evacuation process was not considered in any of the previous works.

The first aim of this paper is to provide a more detailed insight into the properties of macroscopic models of pedestrian motion. In particular, we compare Hughes' model and the second order model analyzing the formation of stop-and-go waves and flows through bottlenecks. Our simulations suggest that Hughes' model is incapable of reproducing neither such waves nor clogging at a narrow exit. It appears to be also insensitive to the presence of obstacles placed in the interior of the walking domain, which can be crucial in the study of evacuation. This is why in the second part of the paper we restrict ourselves only to the second order model.

We focus on the study of the evacuation of pedestrians through a narrow exit. This problem is an important safety issue because of arching and clogging appearing in front of the exit, which can interrupt the outflow and result in crushing of people under the pressure or the crowd. Experimental studies are rare due to the difficulties in reproducing realistic panic behavior [25–27], while numerical simulations are available mainly in the microscopic framework. For example, Helbing et al. [28] and Helbing and Johansson [29] analyzed the evacuation of two hundred people from a room through a narrow door and in [30] the issue of optimal design of walking facilities was addressed with genetic algorithms. At first we show the dependence of the solutions on different parameters of the model. More precisely, we consider the effect on the evacuation of the strength of the interpersonal repealing forces and the desired speed of pedestrians. Both of these parameters may indicate the nervousness and the level of panic of pedestrians.

In order to improve evacuation, Hughes [31] suggested that suitably placed obstacles can increase the flow through an exit. This idea is an inversion of the Braess paradox [32,33], which was formulated for traffic flows and states that adding extra capacity to a network can in some cases reduce the overall performance. In the case of crowd dynamics, placing an obstacle may be seen intuitively as a worse condition. Nevertheless, it is expected to lower the internal pressure between pedestrians and their density in front of the exit and as a result preventing from clogging. This phenomenon has been studied experimentally in case of granular materials by Zuriguel et al. [34] who analyzed the outflow of grains from a silo and found out the optimal height above the outlet of an obstacle which reduces the blocking of the flow by a factor of one hundred. In case of pedestrians, to our knowledge, so far this problem has been studied only numerically. Helbing et al. [5] using the social force model observed that a single column placed in front of the exit decreases the pressure between the column and the door and may prevent from clogging. In the same framework, different shapes and placements of obstacles were studied in [35] with an indication of the formation of the so called "waiting zone" in front of the exit. Frank and Dorso in [36] studied the effects of a column and a longitudinal panel assuming in the social force model that pedestrians change their direction away from an obstacle until the exit becomes visible.

Following the idea of [31], we try to improve the evacuation of pedestrians using properly tuned obstacles placed in front of the exit. Motivated by the numerical simulations in which clogging appears when a large group of pedestrians reached the exit simultaneously, we give an example of a system of five circular columns arranged in the shape of a triangle opened towards the exit. We show that this system of obstacles effectively creates an area with lower density in front of the door and reduces the clogging.

This paper is organized as follows: in Section 2 we explain in detail macroscopic models and in Section 3 we describe numerical approximation of the models. Section 4 is devoted to the numerical results. At first we present error analysis and comparison between the two macroscopic models. Then we analyze the evacuation of pedestrians from a room.