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An approximation analysis for an assembly-like queueing system with time-constraint items

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ABSTRACT

We study an assembly-like queueing system one of whose queues has items with generally distributed time-constraints, where this system has a single server providing services using each item individually. It is well-known that analysis of a queueing system which has items with time-constraint (i.e., impatient items) is difficult since the analytical model must involve all the departure times of these impatient items. We therefore propose to employ the techniques of Whitt's approximation and show the method for obtaining the stationary distribution of the model. Through some simulation experiments, we discuss the validation of our approximation model, and show that the approximation is accurate in various kinds of situations (e.g., service time distribution and the number of queues).

1. Introduction

Many service systems (e.g., automobile manufacturers, pharmaceutical companies, and build-to-order services for personal computers) manufacture products by combining multiple items. Herein items include both demands for service systems as well as items needed to manufacture the end product. These systems contain numerous uncertainties such as mechanical failures during production processes, unplanned maintenance, or delivery delays of items. Because the design and operation of service systems must consider these uncertainties, studies of these systems during the last several decades have extensively employed probability models (queueing models).

Queueing models that provide services by combining multiple items are called *assembly-like queueing systems*. Harrison [1] demonstrated that the assembly-like queueing systems with an infinite capacity are always unstable. Prabhakar et al. [2] showed that the departure process of products weakly converges to a Poisson process if the arrival of each item follows a Poisson process. In these study, the service time is assumed to be instantaneous (i.e., zero). Recently, Alexander et al. [3] studied transient behavior of the departure processes of products, and gave an approximate method for these processes using a Poisson process.

In contrast, assembly-like queueing systems with a finite capacity are always stable. Bhat [4] studied the assembly-like queueing system with Poisson inputs and exponential service times. They gave the waiting time distributions until the service is completed by assuming that the stationary distribution of the queue length of each item can be obtained. Lipper and Sengupta [5] proposed an approximate technique to examine the throughput and mean waiting time by deriving upper and

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lower bounds for the mean waiting time of the model which was similar to [4]. Takahashi et al. [6] generalized one of the arrival processes of the items to a renewal process in which the arrival interval follows a phase-type distribution. They showed that the departure process of products is a Markovian renewal process when the service time is assumed to be zero. Liu and Yuan [7] and Yuan and Liu [8] investigated two-stage tandem queueing systems, including assembly-like queueing systems, where the arrival of items follows a Poisson process and the service times are exponentially distributed. They derived performance measures such as the overflow probabilities of arriving items and those for the shut-down of production due to out-of-stock situations using matrix analytic methods.

In the preceding studies on the assembly-like queueing systems, the effect on the *time-constraint* of the items is not discussed, where the time-constraint means that the item has a deadline until the beginning of its service. The time-constraint can exist prior to a customer receiving services such as when freshness is important, items are affected by trends, or customers' demands have deadlines. The time-constraint in queueing model is known to give a great impact on its performance measures (see, e.g., [9,10] and the references therein). In particular, delays in services may cause cancellations of demands, which can result in potential profit loss as well as damage to the reputation of the service system. Familiar examples that have time-constraint include taxi-cab problem where passengers are impatient, grocery stores which sell fresh foods and build-to-order manufacturing system for computers. Most of these time-constraints can be evaluated using similar models. Hence the study on the assembly-like queueing system with time-constraint items is important.

Furthermore, most of the preceding studies only consider the assembly-like queueing systems with exponentially distributed service times. However, given accidental mechanical failures in production processes and their maintenance times, the characteristics of the system cannot be fully described by assuming the exponential distributions. Thus, our research focuses on analyzing queueing models in which the service time follows a more general distribution, i.e., a phase-type distribution which is known to approximate any probability distribution with arbitrarily desired accuracy (see Chapter III. 4 in [11])).

The main objective of this study is to derive the performance measures of the assembly-like queueing system with time-constraints in an analytically tractable form. However, because of the correlations among queues and the time-constraint of items, the transition structure as a probability model is complicated and depends on the system state. These make both theoretical analysis and numerical calculations difficult.

Specifically, it is difficult to analytically obtain performance measures except for special cases in queueing models with time-constraint items, i.e., impatient items (see, e.g., [9,12–14] and the references therein). To overcome this difficulty, Whitt [15] approximated the structure of the departure process of impatient items by a simple birth-and-death process for an $M/G/s$ queueing model where the impatient time follows a general distribution. In this study, we adopt the approximate technique of [15] to an assembly-like queueing system with time-constraint items, and propose an approximation model which is numerically easy to handle. We further give a computational algorithm for the performance measures of this approximation model, and demonstrate its availability through comparison experiments using simulations. Through some simulation experiments, we discuss the validation of our approximation, and show that the approximation is accurate in various kinds of situations (e.g., service time distribution and the number of queues).

This paper is organized as follows. In Section 2, we introduce the assembly-like queueing system, and explain the approximate technique of [15] for the departure process of impatient items. In Section 3, the approximation model is proposed, and the proposed model is shown to be formulated by a level-dependent quasi-birth-and-death (LDQBD, for short) process. We give a computational algorithm of the stationary distribution for the LDQBD process. The performance measures obtained from the approximation model are given in Section 4. In Section 5, we discuss the validation of the approximation model by comparing with the simulation results.

2. Assembly-like queueing system with time-constraint items

2.1. Description of the models

We consider a queueing system with a single server and $c+1$ waiting lines called queues i for $0 \leqslant i \leqslant c$, where each queue has an input flow of items. When there is no empty queue, an item is drawn from each $c+1$ queues according to first-in first-out (FIFO) discipline in each queue, and is served (*assembled*) at the server. Otherwise, the server remains idle until all queues have at least one item. This queueing system is referred to as an assembly-like queueing system. In this paper, we are interested in the case that items belonging to one of the $c+1$ queues have deadlines for the beginning of their services (see Fig. 1).

We have the following assumptions in this model. For $0 \leqslant i \leqslant c$, items arriving at queue i are referred to as type-i items. The arrival process of type-i ($0 \leqslant i \leqslant c$) items is described by a Poisson process with rate λ_i. The capacity of waiting line for type-0 items is infinite. On the other hand, the size of waiting line (including the waiting space of the server) for type-i ($1 \leqslant i \leqslant c$) item is $\bar{\ell}_i$, where $\bar{\ell}_i$ is a positive integer. Service times at the server are independently and identically distributed with a phase-type distribution having representation $(\boldsymbol{\tau}, T)$ (PH$(\boldsymbol{\tau}, T)$, for short), where $\boldsymbol{\tau}$ is the j_0-dimensional probability row vector and T is the j_0-dimensional ML-matrix (see, e.g., [16]), where j_0 is a positive integer. The phase-type distribution is the distribution of time until a continuous time Markov chain with transition rate matrix T reaches to an absorbing state, given it starts according to initial distribution $\boldsymbol{\tau}$ (see, e.g., [17]). After the service is completed, each item (i.e., $c+1$ different items) is individually consumed and then departs the system.