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Cylindrical bending responses of angle-ply piezoelectric laminates with viscoelastic interfaces

Wei Yan [a,*], J. Wang [b], W.Q. Chen [c]

[a] Faculty of Architectural, Civil Engineering and Environment, Ningbo University, Ningbo 315211, PR China
[b] Faculty of Mechanical Engineering and Mechanics, Ningbo University, Ningbo 315211, PR China
[c] Department of Engineering Mechanics, Zhejiang University, Yuquan Campus, Hangzhou 310027, PR China

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ABSTRACT

The time-dependent behavior of a simply supported, angle-ply piezoelectric laminate in cylindrical bending with viscoelastic interfaces is investigated. The interfacial bonding in piezoelectric laminates is considered to be dielectrically weakly (or highly) conducting, and mechanically compliant characterized by the Kelvin–Voigt viscoelastic law. The state-space approach, which is directly based on the piezoelectricity equations and very effective in analyzing laminated structures, is employed. For exact analysis, a state equation of the relative sliding displacements with respect to the time variable is further presented. Comparison study shows that the numerical results by the present analysis agree well with those reported before. Numerical results also indicate that the electromechanical response of the piezoelectric laminates with viscoelastic interfaces changes remarkably with time elapsing. Thus, the bonding imperfection should be considered carefully in the practical design of piezoelectric laminates.

1. Introduction

Piezoelectric materials have been successfully used as sensors/actuators to control and monitor various engineering structures due to their special properties such as the rapid response and high resolution, the large bandwidth and the little power consumption [1–3]. Thus, the electromechanical behaviors of these so-called smart structures have drawn much attention by many researchers [4–8]. In these mentioned works [4–8], the bonding between two arbitrary adjacent layers was assumed to be perfect. However, some recent investigations have shown that the bonding layer of the piezoelectric structures may significantly affect the local stress distribution between the elastic or piezoelectric layers [9–11].

For a practical laminated structure, it seems very difficult to predict the exact response of the bonding layer either theoretically or experimentally. Some simplified interfacial models thus have been introduced. The most popular one is the linear spring-like model, in which the jump in displacement is linearly proportional to the interfacial traction and the proportional constants are the effective interface parameters (spring-like constant) [12–14]. The shear lag law, which is a special kind of the spring-like model, is usually employed to simulate the connection between the PZT patches and host structures [11,15,16]. In these studies [11–16], the responses of laminated structures under static loading are independent of the time variable. Viscous interfacial models are often introduced artificially to study the time-dependent response of a

* Corresponding author. Tel.: +86 574 87609513; fax: +86 574 87600355.
E-mail address: yanwei4467@sohu.com (W. Yan).

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laminate, especially subjected to the impact loads [17]. He and Jiang [18] derived an exact two-dimensional solution for isotropic laminates with viscous interfaces in cylindrical bending following the famous Pagano's solutions [19,20]. It should be pointed out that the number of unknowns increases significantly with the layer number and hence the exact analysis becomes computationally expensive for a laminate with many layers. A hybrid technology integrated with the state-space method and the power series expansion technique was then employed by Chen and Lee [17] to study the time-dependent behavior of angle-ply laminates in cylindrical bending; and this method has been proven to be very effective and highly accurate. Both works [17,18] showed that, when the time approaches infinity, the viscous interfaces would lose the ability of transferring shear stress totally. This seems however unrealistic for certain types of practical composites, especially within the framework of small deformation. In fact, a viscoelastic interface will be more suitable for simulating the long-term creep and relaxation behavior of the adhesive layer, such as the FRP (Fiber-reinforced polymer)-concrete interfaces [21,22] and the interfaces between the percutaneous medical devices and human skin for long-term implantation [23].

To have a better understanding of the performance of laminates with viscoelastic interfaces, Yan et al. [24] extended Chen and Lee's mixed method [17] to study the responses of a simply-supported laminated orthotropic plate in cylindrical bending with viscoelastic interfaces using the Kelvin–Voigt model [9,25]. It is showed that the main feature of viscoelastic interfaces is that they always keep the function as interlaminar bonds, although weakened. Chen and Lee [17] and Yan et al. [24] concluded that the hybrid analysis, which combining the state-space method with the power series expansion technique converges rapidly for laminates with viscoelastic or viscous interfaces. However, the numerical precision and computational efficiency still rely on the number of total terms and the time step adopted in the power series expansions.

In this paper, the time-dependent behavior of a simply-supported, angle-ply piezoelectric laminate in cylindrical bending with imperfect interfaces is investigated using the state-space method based on the piezoelectricity equations. The interfacial bonding is characterized by the Kelvin–Voigt viscoelastic law mechanically [9,24,25]. Meanwhile, the dielectrically weakly or highly conducting interfaces are also taken into account [26]. The main contribution of the present analysis is to establish a set of differential equations of the relative sliding displacements with respect to the time variable, from which the exact solutions can be derived. This is obviously different from the previous works [17,24], in which the power series expansion technique is employed. Finally, numerical results are presented and discussed.

2. State-space formulation

Consider an N-layered angle-ply piezoelectric laminate under the assumption of cylindrical bending, as shown in Fig. 1. In this case, the displacements u, v and w, in x, y and z directions, respectively, as well as the electric potential ϕ will be independent of the coordinate y. The constitutive relations are [27]

$$\begin{aligned}
&\sigma_x = c_{11}\frac{\partial u}{\partial x} + c_{13}\frac{\partial w}{\partial z} + c_{16}\frac{\partial v}{\partial x} + e_{31}\frac{\partial \phi}{\partial z}, \quad \sigma_y = c_{12}\frac{\partial u}{\partial x} + c_{23}\frac{\partial w}{\partial z} + c_{26}\frac{\partial v}{\partial x} + e_{32}\frac{\partial \phi}{\partial z},\\
&\sigma_z = c_{13}\frac{\partial u}{\partial x} + c_{33}\frac{\partial w}{\partial z} + c_{36}\frac{\partial v}{\partial x} + e_{33}\frac{\partial \phi}{\partial z}, \quad \tau_{xy} = c_{16}\frac{\partial u}{\partial x} + c_{36}\frac{\partial w}{\partial z} + c_{66}\frac{\partial v}{\partial x} + e_{36}\frac{\partial \phi}{\partial z},\\
&\tau_{yz} = c_{44}\frac{\partial v}{\partial z} + c_{45}\frac{\partial u}{\partial z} + c_{45}\frac{\partial w}{\partial x} + e_{14}\frac{\partial \phi}{\partial x}, \quad \tau_{xz} = c_{45}\frac{\partial v}{\partial z} + c_{55}\frac{\partial u}{\partial z} + c_{55}\frac{\partial w}{\partial x} + e_{15}\frac{\partial \phi}{\partial x},\\
&D_x = e_{14}\frac{\partial v}{\partial z} + e_{15}\frac{\partial u}{\partial z} + e_{15}\frac{\partial w}{\partial x} - \varepsilon_{11}\frac{\partial \phi}{\partial x}, \quad D_y = e_{24}\frac{\partial v}{\partial z} + e_{25}\frac{\partial u}{\partial z} + e_{25}\frac{\partial w}{\partial x} - \varepsilon_{12}\frac{\partial \phi}{\partial x},\\
&D_z = e_{31}\frac{\partial u}{\partial x} + e_{33}\frac{\partial w}{\partial z} + e_{36}\frac{\partial v}{\partial x} - \varepsilon_{33}\frac{\partial \phi}{\partial z},
\end{aligned} \tag{1}$$

where $\sigma_i(\tau_{ij})$ and D_i are the normal (shear) stresses and electric displacements, respectively, c_{ij}, e_{ij} and ε_{ij} are the elastic, piezoelectric and dielectric constants, respectively. Just as a usual treatment in viscoelasticity, although the behavior of the laminate will be time-dependent, the deformation is assumed to be very slow so that the inertia terms can be neglected [24]. Thus, the equilibrium equations in absence of body forces and Gaussian equation of electric equilibrium are

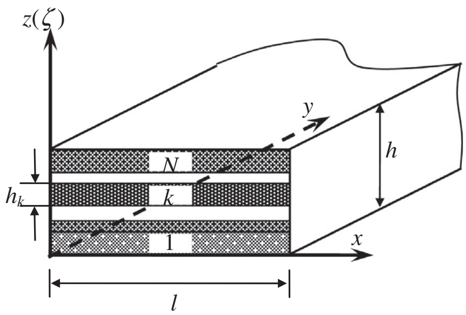

Fig. 1. Sketch of a piezoelectric laminate in cylindrical bending.