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# Comparison of the Adomian decomposition method with homotopy perturbation method for the solutions of seventh order boundary value problems

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ABSTRACT

The aim of this paper is to compare the Adomian decomposition method and the homotopy perturbation method for solving the linear and nonlinear seventh order boundary value problems. The approximate solutions of the problems obtained with a small amount of computation in both methods. Two numerical examples have been considered to illustrate the accuracy and implementation of the methods.



## 1. Introduction

The boundary value problems of ordinary differential equations play an important role in many fields. The theory of seventh order boundary value problems is not much available in the numerical analysis literature. These problems generally arise in modeling induction motors with two rotor circuits. The induction motor behavior is represented by a fifth order differential equation model. This model contains two stator state variables, two rotor state variables and one shaft speed. Normally, two more variables must be added to account for the effects of a second rotor circuit representing deep bars, a starting cage or rotor distributed parameters. To avoid the computational burden of additional state variables when additional rotor circuits are required, model is often limited to the fifth order and rotor impedance is algebraically altered as function of rotor speed. This is done under the assumption that the frequency of rotor currents depends on rotor speed. This approach is efficient for the steady state response with sinusoidal voltage, but it does not hold up during the transient conditions, when rotor frequency is not a single value. So, the behavior of such models show up in the seventh order boundary value problems [1]. Moreover, The mathematical model of synchronous generator can be represented with model of seventh order. This model is complex and it describes the generator accurately [2,3].

Most of the physical phenomena are generally modeled by differential equations along with appropriate conditions. Since the exact solutions to these differential equations are rare so, researchers look for the best approximate solutions. Numerical methods and series solution methods are the tools to find the approximate solutions. Adomian decomposition method and

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homotopy perturbation method are useful frame work for constructing the approximate solutions of boundary value problems.

The Adomian decomposition method (ADM) was introduced and developed by Adomian [4–7]. This method has been applied to a wide class of linear and nonlinear ordinary differential equations, partial differential equations, integral equations and integro-differential equations [8–11]. Wazwaz [11,12] provided the solution of fifth order and sixth order boundary value problems by the modified decomposition method. Convergence of Adomian decomposition method was studied by Cherruault et al. [13–15]. In [16], a proof of convergence of the Adomian's method is presented and the rate of convergence of standard Adomian decomposition method to solve nonsingular and singular ODEs is studied. Al-Hayani [17] presented the Adomian decomposition method with Green's function for the solution of sixth order boundary value problems. The method applied to nonlinear equations does not seem to be fast enough to be an efficient method to solve these kind of equations. The series solution may have small convergence radius.

He [18–22] developed the homotopy perturbation method (HPM) for solving nonlinear initial and boundary value problems by combining the standard homotopy in topology and the perturbation technique. By this method, a rapid convergent series solution can be obtained in most of the cases. Usually, a few terms of the series solution can be used for numerical calculations. Chun and Sakthivel [23] applied homotopy perturbation technique for obtaining approximate solutions of linear and nonlinear second order two-point boundary value problems. Moreover, the authors compared the performance of the homotopy perturbation method with the extended Adomian decomposition method, the exponential fitting method and the shooting method. Sakthivel et al. solved the hyperbolic PDEs with variable coefficients using He's HPM in [24]. Kim and Sakthivel [25] is implemented the $(\frac{G'}{G})$-expansion method to obtain exact solutions for time-delayed Burgers and time- delayed Burgers–Fisher equations. Sakthivel et al. [26] established the new solitary wave solutions for the time-delayed Burgers equation with the help of extended tanh method and the exp-function method. Moreover, the authors calculated the numerical solutions of the time-delayed Burgers equation with initial conditions using the homotopy perturbation method (HPM). The HPM has been successfully applied to ordinary differential equations, partial differential equations and other fields [27,8,18–22,28–30]. The convergence of the homotopy perturbation method was discussed in [31,18,32,33]. The main disadvantage of HPM is that one should suitably choose an initial guess, or infinite iterations are required.

In this paper, the solution of the seventh order boundary value problem is obtained using the Adomian decomposition method and homotopy perturbation method.

## 2. Analysis of the Adomian decomposition method

Consider the following seventh order boundary value problem

$$u^{(7)}(x) = g(x,u) + f(x), \quad a \leqslant x \leqslant b \tag{2.1}$$

with boundary conditions

$$\left.\begin{aligned} u^{(i)}(0) &= \alpha_i, \quad i = 0,1,2,3, \\ u^{(j)}(b) &= \beta_j, \quad j = 0,1,2, \end{aligned}\right\} \tag{2.2}$$

where $u(x)$, $g(x,y)$ is a linear or nonlinear function of $u$, and $f(x)$ is a continuous function defined in the interval $[a,b]$. $\alpha_i,\ i = 0,1,2,3$ and $\beta_j,\ j = 0,1,2$ are finite real constants.

Applying the decomposition method [4–7], Eq. (2.1) can be written as:

$$Lu = f(x) + Nu, \tag{2.3}$$

where $L = \frac{d^7}{dx^7}$ is the linear operator and $Nu = g(x,u)$ is the nonlinear term. Consequently,

$$u(x) = v(x) + \int_a^b G(x,\xi)f(\xi)d\xi + \int_a^b G(x,\xi)Nud\xi, \tag{2.4}$$

where $v(x)$ is the solution of $Lu = 0$ subject to boundary conditions (2.2) and $G(x,\xi)$ is the Green's function given by

$$G(x,\xi) = \begin{cases} g_1(x,\xi), & a \leqslant \xi \leqslant x \leqslant b, \\ g_2(x,\xi), & a \leqslant x \leqslant \xi \leqslant b. \end{cases} \tag{2.5}$$

According to Adomian decomposition method the solution of (2.1) is approximated as an infinite series

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{2.6}$$

and the nonlinear term decomposes as

$$Nu = \sum_{n=0}^{\infty} A_n, \tag{2.7}$$

where $A_n$ are the modified Adomian polynomials defined by