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Exact solution of thermal stress problem of an inhomogeneous hygrothermal piezoelectric hollow cylinder

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ABSTRACT

The closed-form exact solution for the hygrothermal response of inhomogeneous piezoelectric hollow cylinders is obtained. The interaction of electric potentials, electric displacement and elastic deformations is presented. The present cylinder is subjected to both a mechanical load and an electric potential. The material properties coefficients of the present cylinder are assumed to be changed in the radial direction by different distribution forms. The field quantities like displacement, stresses and electric potentials in the inhomogeneous piezoelectric cylinders are determined. The significant of influences of material inhomogeneity, initial temperature, final moisture, and the load and electric ratios in the field quantities are investigated. The concluding remarks and suitable discussions are made.

1. Introduction

The stresses and strains in composite structures subjected to mechanical and hygrothermal loads have been proposed in numerous analyses in the past and still the subject of research interest in recent years. Most structures are usually subjected to changing environmental conditions during both initial fabrication and final use. The effect of temperature is known as thermal effect and the effect of moisture absorption from the atmosphere is known as hygroscopic effect. The combined effect of temperature and moisture is known as hygrothermal effect. Heat gets conducted into the laminate when subjected to rise in the temperature. The laminate absorbs moisture when subjected to the wet conditions. The swelling or expansion is more across the fibers of the lamina. Hygrothermal effects induce a dimensional change in the laminate. But due to the mismatch of the properties of the constituents of the laminate, its free movement is inhibited. As a result, deformations and corresponding stress conditions are induced. The induced hygrothermal stresses are referred as residual stresses.

Studies on the effects of temperature and moisture, individually and combined, on composite materials has been carried out somewhat extensively. However, the effects of the continuous alternation of the two conditions, especially concurrently with mechanical loading, have been less explored. Lee and Yen [1] have presented the nonlinear problem of moisture and temperature effects on the stability analysis of orthotropic cylindrical shell panels subjected to axial or in-plane loading using finite element method. A linear theory of coupled heat and moisture is adopted by Chang et al. [2] to analyze the transient responses in an infinitely long annular cylinder subjected to hygrothermal loadings. Kollár et al. [3] have applied

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the stress analysis to fiber-reinforced organic matrix composite cylinders subjected to hygrothermal and mechanical loads. Chang [4] has used a decoupling technique to obtain analytical solutions for the heat and moisture diffusion occurring in a solid cylinder subjected to hygrothermal loadings. Kollár [5] has developed the response of fiber-reinforced composite cylinders to axially varying mechanical and thermal loads. The influence of hygrothermal effects on the buckling and post-buckling of shear deformable laminated plates and cylindrical shells subjected to combined loading of axial compression and external pressure are investigated by Shen [6,7]. Sayman [8] has developed a general stress analysis for thick or thin multi-layered composite cylinders under hygrothermal loadings. Dağhan [9] has studied the effects of combined internal pressures, temperatures, and moistures on filament-wound multilayered composite cylinders for the plane-strain case. Nosier and Miri [10] have studied the free-edge effects in laminated, circular, cylindrical shell panels subjected to hygrothermal loading by utilizing displacement-based technical theories. Akbarzadeh and Chen [11] have obtained the magnetoelectroelastic responses of radially polarized and magnetized hollow and solid cylinders subjected to hygrothermal loading by a straightforward analytical method.

A class of piezoelectric structures has been developed in recent decades for use in various engineering applications. Piezoelectric materials have been widely used in many industrial applications due to the special electromechanical coupling effects. Many theoretical investigations have been reported in the analysis for piezoelectric composite structures with perfectly bonded interfaces. The topics include static analysis, free vibration, wave propagation and transient response. Kharouf and Heyliger [12] have presented a numerical method for finding approximate solutions to static and axisymmetric vibrations problems for homogeneous and laminated piezoelectric cylinders. Hou et al. [13] have proposed an analytical method to solve the axisymmetric plane strain electro-elasto-dynamic problem of a special non-homogeneous piezoelectric hollow cylinder subjected to dynamic loads. Dai and Wang [14] have presented an analytical solution for the interaction of electric potentials, electric displacements, elastic deformations and mechanical loads, and describes electro-magneto-elastic responses and perturbation of the magnetic field vector in a piezoelectric hollow cylinder subjected to sudden mechanical load and electric potential. Wang et al. [15] have obtained the transient responses in a two-layered elasto-piezoelectric composite hollow cylinder in the state of axisymmetric plane strain. Dai et al. [16] have presented the analytical study for electro-magneto-thermo-elastic behaviors of a hollow cylinder composed of functionally graded piezoelectric material, placed in a uniform magnetic field, subjected to electric, thermal and mechanical loads. Wang [17] has investigated the dynamic electromechanical behavior of a triple-layer piezoelectric composite cylinder with imperfect interfaces. Fesharaki et al. [18] have considered a hollow cylinder made of functionally graded piezoelectric material subjected to non-axisymmetric electrical and mechanical loads. Wu and Tsai [19] have presented the 3D coupled analysis of simply-supported, functionally graded piezoelectric material circular hollow sandwich cylinders under electro-mechanical loads. Zenkour [20] has presented the piezoelectric behavior of an inhomogeneous hollow cylinder with thermal gradient. Akbarzadeh and Chen [21] have presented analytical solutions for hygrothermal stresses in 1-D functionally graded piezoelectric media.

In this investigation, both ambient temperature and moisture are assumed to have variable distributions through the thickness of the cylinder. The material properties, piezoelectric parameters, dielectric parameter and pyroelectric coefficients are assumed to be functions of the radial direction of the cylinder. The hygrothermoelastic response of an inhomogeneous piezoelectric hollow cylinder is presented. The effects of various parameters on the field quantities are investigated.

2. Hygrothermoelastic response of a piezoelectric cylinder

Consider a long inhomogeneous hollow cylinder of inner radius a and outer radius b and having perfect conductivity. Let the cylindrical coordinates of any representative point be (r, θ, z) and assume that the cylinder is subjected to radially changing of temperature $T(r)$ and moisture concentration $C(r)$. For the axisymmetric plane strain assumption, the components of displacement, stresses, and electric displacement and electric potential may be expressed as $u(r)$, $\alpha_i(r)$, $D_r(r)$ and $\psi(r)$, respectively. The constitutive relations are:

$$\begin{Bmatrix} \sigma_r \\ \sigma_\theta \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \left(\begin{Bmatrix} \frac{du}{dr} \\ \frac{u}{r} \end{Bmatrix} - \begin{Bmatrix} \alpha_1 \\ \alpha_2 \end{Bmatrix} T - \begin{Bmatrix} \beta_1 \\ \beta_2 \end{Bmatrix} C \right) + \begin{Bmatrix} e_{11} \\ e_{12} \end{Bmatrix} \frac{d\psi}{dr}, \tag{1}$$

$$\sigma_z = c_{13}\frac{du}{dr} + c_{23}\frac{u}{r} + e_{13}\frac{d\psi}{dr}, \tag{2}$$

and

$$D_r = e_{11}\frac{du}{dr} + e_{12}\frac{u}{r} - \varepsilon_{11}\frac{d\psi}{dr} + p_{11}T + p_{22}C, \tag{3}$$

where c_{ij} are the elastic coefficients, e_{1j} are the piezoelectric parameters, ε_{11} is the dielectric parameter, p_{11} and p_{22} are the pyroelectric coefficients, and α_i and β_i are the thermal and moisture expansion coefficients. Let the coefficients c_{ij}, e_{1j}, ε_{11}, p_{11} and p_{22} are changed through the radial direction of the present hollow cylinder according to the following relation:

$$P(r) = P^0\left(\frac{r}{b}\right)^{-2n}, \quad a \leqslant r \leqslant b, \tag{4}$$