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Short communication

Global residue harmonic balance method to periodic solutions of a class of strongly nonlinear oscillators

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ABSTRACT

A new approach, namely the global residue harmonic balance method, was advanced to determine the accurate analytical approximate periodic solution of a class of strongly nonlinear oscillators. A class of nonlinear jerk equation containing velocity-cubed and velocity times displacements-squared was taken as a typical example. Unlike other harmonic balance methods, all the former residual errors are introduced in the present approximation to improve the accuracy. Comparison of the result obtained using this approach with the exact one and simplicity and efficiency of the proposed procedure. The method can be easily extended to other strongly nonlinear oscillators.

1. Introduction

The study of nonlinear problems is of great importance in all areas of Physics and Engineering. It is very difficult to solve nonlinear problems and it is often more difficult to get an analytical approximation than a numerical one to a given nonlinear problem. Considerable attention has been paid towards the study of strongly nonlinear oscillators and many methods have been used to find approximate solutions to nonlinear oscillators [1–15]. In recent years, the nonlinear jerk equations involving the third temporal derivative of displacement have been widely studied [1–13]. As well as describing many mechanics, nonlinear jerk equations are finding increasing importance in the study of physical problems, such as chaos [9–11].

Recently, many approximate methods were presented to determine analytical and approximate solutions to the nonlinear jerk equations. Gottlied [1,13] has explored the flexibility of applying the method of harmonic balance to achieve analytical approximate periodic solutions of nonlinear jerk equations. Wu et al. [2] proposed an improved harmonic balance method for determining the periodic solutions of nonlinear jerk equations, and their second and third approximations give accurate results for a large range of the initial velocity amplitude. Ma et al. [3] and Hu et al. [4] applied, respectively, homotopy perturbation and parameter perturbation to the jerk equations. Hu et al. [5] generalized the Mickens iteration procedure to determine the periodic solution of nonlinear jerk equations. Ramos [6] obtained an accurate result by means of a second Linstedt-Poincare method. Feng and Li [7] applied homotopy analysis to determine a class of jerk equation. Recently, Leung et al. [8] used residue harmonic balance method to solve nonlinear jerk equations, and they obtained some more accurate results. Many researchers have shown that residual error exerts a significant effect on the approximation accuracy [14,15]. But the residue harmonic balance method did not take all the residual error in their process of calculating the solutions.

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In this paper, we put forward a novel approximate method, namely the global residue harmonic balance method, to determine the periodic solutions of nonlinear jerk equations. To obtain higher-order analytical approximations, all the residual errors are considered in the process of every order approximation. The technique is different from the residue harmonic balance which does not use all the residue. We can see this by the process of solution and a later example results, which are given to illustrate the applicability and accuracy of the technique.

2. Basic ideas of the global residue harmonic balance approach

For the sake of simplicity, we have been considered systems governed by equations having the form

$$\Phi(\dddot{u}, \ddot{u}, \dot{u}, u) = 0, \quad u(0) = A, \quad \dot{u}(0) = 0, \tag{1}$$

where the over-dot denotes differentiation with respect to t. For convenience, we assume Eq. (1) is a conservative system [i.e. $\Phi(-\dddot{u}, -\ddot{u}, -\dot{u}, -u) = -\Phi(\dddot{u}, \ddot{u}, \dot{u}, u)$].

Suppose ω is the angular frequency of the Eq. (1) to be determined. With a new independent variable $\tau = \omega t$, Eq. (1) becomes

$$\Phi(\omega^2 u''', \omega u'', \omega u', u) = 0, \quad u(0) = A, \quad u'(0) = 0, \tag{2}$$

where prime denotes the derivative with respect to τ.

Considering the periodic solution does exist, it may be better to approximate the solution by such a set of base functions

$$\{cos((2k-1)\tau) | k = 1, 2, 3, \ldots\}. \tag{3}$$

According to Eq. (3), the initial approximate periodic solution satisfying initial conditions in Eq. (2) is

$$u_0(\tau) = Acos(\tau), \quad \tau = \omega_0 t, \tag{4}$$

where ω_0 is an unknown constant to be determined later.

Substituting Eq. (4) into Eq. (2), it results the following residual

$$R_0(\tau) = \Phi(\omega_0^2 u_0''', \omega_0 u_0'', \omega_0 u_0', u_0). \tag{5}$$

If $R_0(\tau) = 0$, then $u_0(\tau)$ happens to be the exact solution. Generally such case will not arise for nonlinear problems.

The left hand side of Eq. (5) should not contain secular terms of $cos(\tau)$. Equating its coefficients to zero, we can determine the unknown constant ω_0. Then, the zero-order approximation u_0 is in the form of Eq. (4).

Based on the zero-order approximation Eq. (4) and the residual Eq. (5), we consider the following approximate periodic solution and frequency

$$u(\tau) = u_0(\tau) + pu_1(\tau), \quad \omega^2 = \omega_0^2 + p\omega_1, \tag{6}$$

where p is the order parameter with values in the interval [0, 1].

Substituting Eq. (6) into Eq. (2) and equating the coefficients of the p, we can get

$$F_1(\tau, \omega_1, u_1(\tau)) \triangleq \left(\omega_1 \frac{\partial}{\partial(\omega^2)} + u_1''' \frac{\partial}{\partial u'''} + u_1'' \frac{\partial}{\partial u''} + u_1' \frac{\partial}{\partial u'} + u_1 \frac{\partial}{\partial u}\right)\Phi_0, \tag{7}$$

where $\partial\Phi_0/\partial u$ denotes that $\partial\Phi/\partial u$ is to be evaluated at the zero-order approximation after differentiation etc. It is noted that Eq. (7) is linear with respect to ω_1 and u_1. Noting that the solution has the form of Eq. (3), we choose

$$u_1(\tau) = a_{3,1}(cos(\tau) - cos(3\tau)). \tag{8}$$

Substituting Eq. (8) into Eq. (7), we consider the following equation

$$F_1(\tau, \omega_1, u_1(\tau)) + R_0(\tau) = 0. \tag{9}$$

All the residual errors of the zero-order approximation $R_0(\tau)$ are introduced into Eq. (9) to improve the accuracy.

The right hand side of Eq. (9) should not contain the terms $cos(\tau)$ and $cos(3\tau)$ based on Galerkin technique. Letting their coefficients be zeros, we obtain two linear equations containing two unknowns ω_1 and $a_{3,1}$. Then the two unknown constants can be solved easily. Thus, we get the first-order approximation

$$u_{(1)}(\tau) = u_0(\tau) + u_1(\tau), \quad \omega_{(1)}^2 = \omega_0^2 + \omega_1, \quad \tau = \omega_{(1)} t, \tag{10}$$

where $u_0(\tau)$ and $u_1(\tau)$ are given by Eqs. (4) and (8) respectively.

Substituting Eq. (10) into Eq. (2), it results the following residual

$$R_1(\tau) = \Phi(\omega_{(1)}^2 u_{(1)}''', \omega_{(1)} u_{(1)}'', \omega_{(1)} u_{(1)}', u_{(1)}). \tag{11}$$

Owing to the nonlinearity, $R_1(\tau)$ is in general nonzero. We will construct higher-order approximations based on the residual errors.