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## Models of impulsive culling of mosquitoes to interrupt transmission of West Nile virus to birds

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### ABSTRACT

A mathematical model describing the transmission of West Nile virus (WNV) between vector mosquitoes and birds, incorporating a control strategy of culling mosquitoes and defined by impulsive differential equations is presented and its properties investigated. First, we consider a strategy of periodic impulsive culling of the mosquitoes. Theoretical results indicate that if the threshold  $R_0$  is greater than unity the disease uniformly persists, but, if not, the disease does not necessarily become extinct. The explicit conditions determining the backward or forward bifurcation were obtained. The culling rate has a major effect on the occurrence of backward bifurcation. Analysis shows that the disease is most sensitive to mosquito–bird contacts, mosquito–culling rate and intervals between culls. The dependence of the outcomes of the culling strategy on mosquito biting rate is discussed. When the complete elimination of disease is impossible, mosquito culls are implemented once the infected birds reach a predefined but adjustable threshold value. Numerical analysis shows that the period of mosquito culling finally stabilizes at a fixed value. In addition, variations of mean prevalence of WNV in birds and the culling period are simulated.

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## 1. Introduction

West Nile virus (WNV) is a virus of the family Flaviviridae. It mainly infects birds, and it is also known to infect humans, horses, dogs, cats, bats, chipmunks, skunks, squirrels, domestic rabbits, crocodiles and alligators [1–6]. In the USA, the disease is of serious public-health importance, with more than 30,000 cases and hundreds of deaths reported in 48 States since 1999 [7]. WNV is maintained in nature in a mosquito–bird–mosquito cycle [8–10], but the disease can be passed on to humans as a zoonotic disease when an infected mosquito changes hosts to bite people, who are dead-end hosts. The principal vectors are mosquitoes in the genera *Culex*, *Aedes*, *Anopheles* and *Ochlerotatus*, but other genera are also known to be infected in the wild. In the absence of an effective vaccine and/or treatment, anti-WNV efforts are primarily based on mosquito-reduction strategies (such as larviciding, adulticiding and elimination of breeding sites) and personal protection (based on the use of appropriate insect repellents). These measures are intensified during mosquito seasons [4,11].

Compartmental epidemiological models have played a significant role in understanding the mechanisms of dynamical transmission of WNV. Lewis et al. studied the existence of travelling waves describing the speed of the spatial spread of the virus [12]. Lewis et al. also made a comparative study of discrete-time and continuous-time models to investigate

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WNV transmission [13]. Bowman et al. formulated a model system incorporating mosquito-bird-human populations for assessing control strategies against WNV [5]. In addition, ordinary differential equation models have been analyzed to investigate the threshold conditions for WNV outbreaks with backward bifurcation [2–4]. However, most of these mathematical models considering control measures on mosquitoes invariably assume that the pesticides affect mosquitoes continuously, but usually mosquito culling takes place only at certain times. It is known that impulsive differential equations can be used to describe pesticide sprays and analyse pest control strategies [14–19]. As mosquito culling is a common method for WNV control [14,15], we adopt it in this paper. The birds' population is considered to vary with time [3,4,8,10], compared with the constant assumption in [14,15]. Therefore our main purposes are to investigate the transmission of WNV between bird and mosquito populations with impulsive control strategies; analyze the dynamical behavior theoretically and investigate phenomena introduced by impulsive culling; determine the most rational strategy to control the transmission of WNV; and finally find out the mosquito-culling period to keep the infected mosquitoes always less than the adjustable threshold when the complete elimination of disease is impossible.

To achieve the above goals, we formulate two mathematical models, considering periodic or state-dependent pesticide sprays as control measures, to investigate the transmission of WNV between mosquitoes and birds, where the total number of birds varies with time. First, we propose impulsive differential equations, which have already been used to investigate malaria in human-mosquito populations [20], to describe the process of periodic culling of mosquitoes. Similar methods can be found in [21–26]. Conditions for persistence of the disease and the occurrence of backward bifurcation are obtained theoretically. In order to assess the control strategy, sensitivity analysis is applied to study the contribution of each parameter on the disease transmission. If complete eradication of WNV is not possible, we extend our equations to a state-dependent model. The positive periodic solution with the conditions for the maximum value no more than the critical threshold is obtained numerically with a relatively high bird-mosquito contact rate. In addition, by considering resource savings and environmental protection, we can change the state-dependent impulsive control problem into a fixed-time impulsive control problem.

**2. WNV control with fixed moments**

We first consider the strategy of implementing periodic culling of mosquitoes at critical times, with differential equations proposed as follows. The total female mosquito population at time  $t$ , denoted by  $N_m(t)$ , is split into the populations of susceptible ( $S_m(t)$ ) and infected ( $I_m(t)$ ) mosquitoes. The susceptible mosquito number is increased via births or immigration at a constant rate  $\Lambda_m$  and diminished by infection – which may be acquired when uninfected mosquitoes feed from the blood of infected birds – and by death due to natural causes at a rate  $\mu_m$ . The infected mosquito number is generated via the infection of susceptibles and diminished by natural deaths at a rate  $\mu_m$ .  $T > 0$  represents the mosquito-culling (such as spraying) interval. We assume that spraying reduces both susceptible and infected mosquitoes and  $0 \leq p \leq 1$  is the proportion of those mosquitoes killed [20]. Similarly, the total bird population at time  $t$ , denoted by  $N_b(t)$ , is split into the populations of susceptible ( $S_b(t)$ ) and infected ( $I_b(t)$ ) birds.

$$\left. \begin{aligned} \frac{dS_m}{dt} &= \Lambda_m - c\beta_{mb} \frac{I_b}{N_b} S_m - \mu_m S_m, \\ \frac{dI_m}{dt} &= c\beta_{mb} \frac{I_b}{N_b} S_m - \mu_m I_m, \\ \frac{dS_b}{dt} &= \Lambda_b - c\beta_{bm} \frac{S_b}{N_b} I_m - \mu_b S_b, \\ \frac{dI_b}{dt} &= c\beta_{bm} \frac{S_b}{N_b} I_m - (\mu_b + d_b) I_b \end{aligned} \right\} \quad t \neq nT, \quad n \in \mathbb{N}, \quad \mathbb{N} = 1, 2, \dots \tag{1}$$

$$\left. \begin{aligned} S_m(t^+) &= (1 - p)S_m(t), \\ I_m(t^+) &= (1 - p)I_m(t). \end{aligned} \right\} \quad t = nT, \quad n \in \mathbb{N}, \quad \mathbb{N} = 1, 2, \dots$$

where  $c$  is the average biting rate of the mosquitoes,  $\Lambda_b$  is the recruitment rate of birds,  $\beta_{bm}$  and  $\beta_{mb}$  are the transmission probabilities of WNV from mosquitoes to birds and from birds to mosquitoes respectively,  $\mu_b$  is the natural death rate of the birds, and  $d_b$  is the WNV-induced death rate. All parameters are defined in Table 1.

**3. Existence and stability of the disease-free periodic solution**

First we consider the subsystem of (1) in the disease-free subspace  $X_s = \{(S_m, I_m, S_b, I_b) : S_m \geq 0, I_m = 0, S_b \geq 0, I_b = 0\}$  as follows

$$\left\{ \begin{aligned} \frac{dS_m}{dt} &= \Lambda_m - \mu_m S_m, \\ \frac{dS_b}{dt} &= \Lambda_b - \mu_b S_b, \end{aligned} \right\} \quad t \neq nT, \quad n \in \mathbb{N}, \tag{2}$$

$$S_m(t^+) = (1 - p)S_m(t), \quad t = nT, \quad n \in \mathbb{N}.$$

Note that the bird population is free from impulse, so we know  $S_b(t) \rightarrow \Lambda_b/\mu_b$ , as  $t \rightarrow \infty$ . Without loss of generality, we let  $\bar{S}_b(t) = \Lambda_b/\mu_b$ . The equations for the mosquitos in (2) are similar to those in [20], in which the decoupled impulsive differ-

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