Applied Mathematical Modelling 39 (2015) 3569–3587

Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

On the torsional wave dispersion in a hollow sandwich circular cylinder made from viscoelastic materials

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ARTICLE INFO

Article history:
Received 27 October 2013
Received in revised form 24 September 2014
Accepted 24 November 2014
Available online 11 December 2014

Keywords:
Attenuation
Dispersion
Sandwich hollow cylinder
Viscoelastic material
Torsional waves

ABSTRACT

The dispersion of torsional wave propagation in the sandwich hollow cylinder made of linear viscoelastic materials is investigated. The investigations are carried out within the scope of the piecewise homogeneous body model by utilizing the exact field equations of viscoelasto-dynamics. The mechanical relations of the materials are described through fractional exponential operators. The analytical expression for the dispersion equation is obtained and the algorithm is developed for its numerical solution. The influence of the viscosity of the layers of the sandwich hollow cylinder is studied through the rheological parameters which characterize the characteristic creep time and long-term values of the elastic constants. It is assumed that the materials of the inner and outer cylinders are the same. Numerical results (dispersion curves) are obtained and the influence of the mentioned rheological parameters on these curves is investigated for the following two cases: attenuation of the torsional waves is dispersive (Case 1) and attenuation of the torsional waves is non dispersive (Case 2). The analytical expression is obtained for the low wavenumber limit values of the torsional wave propagation velocity which is valid in Case 1. As a result of the numerical investigations, in particular, it is established that in the case where the rheological parameters of the layers are the same, the viscosity of the layers' materials causes the torsional wave propagation velocity to increase.

1. Introduction

The study of the propagation (dispersion) and attenuation of time-harmonic waves in viscoelastic materials and in elements of constructions such as rods, plates etc., made from these materials has great significance in the fundamental (theoretical), as well as in the application sense. One such application is the nondestructive inspection of tubes and pipes which are used in the infrastructure of many industries such as gas, oil, and water transport. Note that in many cases such tubes are coated with viscoelastic polymer coatings for corrosion protection. Therefore, under nondestructive testing of these

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http://dx.doi.org/10.1016/j.apm.2014.11.061
0307-904X/

tubes with guided waves, it is necessary to know the attenuation and dispersion rules of the waves in the sandwich tubes with the viscoelastic layers to obtain applicable and sufficiently accurate results.

Another application of the study of the propagation of guided waves in viscoelastic bodies is the use of the viscoelastic systems for attenuation of vibrations and waves caused by an earthquake or with the various types of sound sources.

Nevertheless, up to now investigations related to wave propagation in structural elements (such as plates, cylinders etc.) made from viscoelastic materials are not as high as related studies which have been made for the same structural elements made from purely elastic materials.

Here we consider a brief review of some of them and begin with the papers by Weiss [1] and Tamm and Weiss [2] which relate to the Lamb wave propagation in an isotropic viscoelastic layer with stress-free surfaces. Note that in these papers it is assumed that the elastic constants are complex and independent of frequency. An approximate method for investigation of the Lamb wave propagation in a plate from viscoelastic materials with small losses and frequency-dependent elastic moduli was proposed by Coquin [3]. Numerical techniques for investigation of the influence of low-compressibility materials with real Poisson's ratio and frequency dependent complex shear moduli on the propagation of Lamb waves was developed by Chervingo and Sevchenko [4].

Lamb wave propagation in elastic plates coated with viscoelastic materials was studied by Simonetty [5] and the effect of attenuative coatings on the dispersion characteristics of Lamb waves in the elastic plate was analyzed. The results obtained in the aforementioned papers were also noted and some of them are detailed in the monograph by Rose [6].

Axisymmetric non-stationary torsional wave propagation in the semi-infinite circular cylinder was considered in the paper by Wolosewick and Raynor [7], in which it was assumed that an arbitrary radial axially symmetric tangential shear stress distribution harmonic in time acts on the end of the cylinder.

In a paper by Barshinger and Rose [8] axisymmetric guided wave propagation in an elastic hollow cylinder coated with a viscoelastic material was studied. Numerical results on the phase velocity dispersion and attenuation dispersion were presented and discussed.

As follows from the analyses of the foregoing papers, the investigations on the dispersion of the guided waves in the plates or cylinders made from viscoelastic materials were carried out mainly in the following cases: (a) the complex modulus of viscoelastic materials is taken as frequency independent; (b) the viscoelasticity of the materials are described by the simplest models such as the Maxwell and Kelvin–Voigt models, or the expression for the complex elasticity modulus obtained experimentally for the concrete polymer material is used.

Consequently, in the works reviewed above the corresponding investigations on wave dispersion were not connected with the more complicated and real models for viscoelastic materials and a few of the numerical results obtained in these works do not illustrate the influence of the rheological parameters of the viscoelastic materials on this dispersion.

In the present paper we attempt to investigate the torsional wave propagation and dispersion in the sandwich hollow cylinder made from linear viscoelastic materials, the mechanical relations for which are given through the fractional exponential operators by Rabotnov [9] and the numerical results obtained for the wave dispersions and attenuation dispersion are connected with the rheological parameters which enter these operators. Note that the mentioned fractional-exponential operators allow us to describe, with the very high accuracy required, the initial parts of the experimentally and theoretically constructed creep and relaxation graphs and their asymptotic values. Therefore these operators are employed successfully to describe various polymer materials and epoxy-based composites with continuous fibers or layers. Through variation of the rheological parameters contained within these operators many possible cases can be considered which relate to the dynamics of the viscoelastic materials. At the same time, these operators have many simple rules for complicated mathematical transformations, for example, the Fourier and Laplace transformations which we will use in the present investigation.

2. Formulation of the problem

We consider the sandwich hollow circular cylinder shown in Fig. 1 and assume that the radius of the internal circle of the inner hollow cylinder is R and the thickness of the inner, middle and outer cylinders is $h^{(1)}$, $h^{(2)}$ and $h^{(3)}$, respectively. The values related to the inner, middle and external hollow cylinders will be denoted by the upper indices (1)–(3), respectively.

We assume that the materials of the constituents are isotropic, homogeneous and hereditary-viscoelastic. We use the cylindrical system of coordinates $Or\theta z$ (Fig. 1) for determination of the position of points of the system under consideration. Moreover, we assume that the cylinders have infinite length in the direction of the Oz axis.

Thus, within the scope of the piecewise homogeneous body model let us investigate the axisymmetric torsional wave propagation along the Oz axis in the considered body with the use of the equations of motion of the linear theory for viscoelastic bodies.

We write the governing equations and mechanical relations for the case under consideration.

The equation of motion is:

$$\frac{\partial}{\partial r}\sigma_{r\theta}^{(n)} + \frac{\partial}{\partial z}\sigma_{z\theta}^{(n)} + \frac{2}{r}\sigma_{r\theta}^{(n)} = \rho^{(n)}\frac{\partial^2}{\partial t^2}u_{\theta}^{(n)}. \tag{1}$$